Cognitive Maps: an Overview and their Application for Student Modeling

Mapas Cognitivos: un Perfil y su Aplicación al Modelado del Estudiante

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Abstract
In this paper we state how Cognitive Maps can be used to model causal phenomena. In addition, we show the application of the Cognitive Maps to the field of the Student Modeling. Conceptually speaking, Cognitive Maps set and simulate the systems dynamics based upon qualitative knowledge. A Cognitive Map is a tool that gives away the entities of the issue of study. Moreover, Cognitive Maps bring out the causal phenomena as cause-effect relationships between concepts. According to the relationships, a topology and a workflow of causal effects is designed. Cognitive Maps aim to predict the evolution of a model through simulation. During the process are achieved causal inferences that estimate the variation on the state of the concepts. The simulation breaks down when the concept values reach a fixed point, a pattern of states or a chaotic region in the search space. Wherefore, in this paper we depict the underlying concepts for Causal Modeling by means of Cognitive Maps. In addition, three versions of Cognitive Maps are outlined. Besides to reveal their mathematical baseline, we illustrate their application through the development of a case of study focus on Student Model.
Keywords: Cognitive Maps, Causal-effect relationships, Concepts, Causal inference, and Student Model.

Resumen
En este artículo se establece como usar los Mapas Cognitivos para modelar fenómenos causales. Además, mostramos su aplicación en el Modelado del Estudiante. Conceptualmente hablando, los Mapas Cognitivos definen y simulan la dinámica de sistemas por medio de conocimiento cualitativo. Un Mapa Cognitivo es una herramienta que revela las entidades del objeto de estudio. Así mismo, los Mapas Cognitivos expresan el fenómeno causal como relaciones causa-efecto entre conceptos. De acuerdo con las relaciones, una topología y un flujo de efectos causales es diseñada. Los Mapas Cognitivos buscan predecir la evolución del modelo mediante simulación. Durante el proceso se realizan inferencias que estiman la variación del estado de los conceptos. La simulación termina cuando los valores de los conceptos arriban a punto fijo, a un patrón de estados, o a una región de caos en el espacio de búsqueda. Por tanto, en este artículo se definen los conceptos base para el modelado causal a través de Mapas Cognitivos. También se presentan tres versiones de Mapas Cognitivos. Además se expresa la base matemática y se ilustra su aplicación en el desarrollo de un Modelo del Estudiante.
Palabras clave: Mapas Cognitivos, Relaciones Causales, Conceptos, Inferencia Causal, y Modelo del Estudiante.

1 Introduction

Cognitive Maps (CM's) is a term with a broad meaning that has been used to focus on specific targets, as the ones based on cause-effect relationships. The interest about *Causal CM's* is that the causality is a post hoc explanation of real world events. The baseline of the causality rest on the philosophical principle that states: Any fact has a cause, and given the same conditions, the same causes produce the same consequences [Carvalho, 2001]. The earliest version of Causal CM's corresponds to the model proposed by Axelrod in 1976. In that time, Axelrod outlined an international affairs issue through the sketch of pure causal links between concepts [Peña and Gutierrez, 2004]. Since then a broad spectrum of applications has been carried out in many fields like: Decision support [Nakamura et al., 1982], Multi-Agent Systems [Chaib-draa, 2002], virtual reality [Dickerson and Kosko, 1997], Probabilistic causality [Wellman, 1994], On-Line Analytical Process [Zhang, 2003], and much more [Aguilar, 2003]. Nowadays Causal

CM's research is working on systems dynamics, automatic generation and integration with other families of CM's [Peña et al., 2005[a]], [Peña et al., 2005[b]].

In order to provide an overview of the CM's, besides to show the baseline of three CM's versions and depict how to use them for Student Modeling, the organization of the paper is as follows: In second section the Conceptual Model of the CM's is introduced. Sections 3, 4, and 5, are devoted to set the formal model for the Qualitative, Fuzzy, and Rule-Base Fuzzy CM's versions. Moreover, a Student Model application is carried out from those perspectives. In section 6 we discus the advantages and weakness of the three versions of CM's, besides of outline the further work.

2 Cognitive Maps Profile

This section is oriented to introduce the Conceptual Model for the CM's that sets the underlying elements behind any version. In addition, the components that shape the cognitive mapping process are stated. So the first part is dedicated to present a proposal of an underlying model for the CM's. Moreover, key concepts about qualitative knowledge, causal reasoning, and systems dynamics are introduced. Afterwards, the second section is devoted to give away the elements used to sketch CM's, as: Graphs, values for the concepts, types of relationships and causal effect's estimation.

2.1 Underlying Concepts

In short, a CM is a graphical mental model that externalizes as a person understands, believes and organizes a subject of analysis. So a CM is a partial, incomplete and monotonic representation of the individual's point of view. This type of perspective pursues to reveal the thoughts of the individual through concepts, relationships and inferences. A CM points out the entities that correspond to objects or phenomena from the context of study, and it calls them as *concepts*. What is more, a CM states the individual's beliefs about how a given concept is the responsible for the perturbation on the state of another concept. These kinds of judgments are known as *cause-effects relationships* between concepts. According to the experience, analogy, intuition and common sense of the individual, *causal inferences* are carried out to estimate behavior and outcomes. The conclusions stemmed are exclusively considered to be true and valid in the context of analysis.

Concepts, cause-effect relationships and causal inference compose a kind of qualitative knowledge. Such mental repository is conformed by a set of judgments. These kinds of thoughts are outcome in everyday life at the moment individuals are surrounded by abstract and physical objects and phenomena. The properties of these entities shape the consciousnesses of the individuals and reveal the predicate of their judgments. According to the perceptions and beliefs of the individuals, the links of their judgments are established for outlining their thoughts. The baseline of such judgments is the cause-effect belief. Causality deals with events that are incidents that happen usually due to some reasons.

A CM is the result of a cognitive process called: *cognitive mapping*. During this process the individual chooses the concepts that point out the issue to be analyzed. The variety and level of detail of the concepts lies on the interests of the person who carries out the CM model. Also, the process applies the causality for outlining the relationships between one or more *cause* events, that conjunctively or disjunctively, are able to trigger the occurrence of a *consequence* event according to a specific context of analysis.

Causal reasoning is based upon the general definition for reasoning, inference and incomplete induction stated by Miguelena (2000). The reasoning is the kind of thought that takes one or more well-known judgments that are logically joined in order to yield a new judgment stated as conclusion. The reasoning uses the inference process to achieve a conclusion from a set of premises. The causal inference rests on inductive reasoning for arriving to a universal conclusion from a set of individual or partial judgments. However, the induction is incomplete as causal reasoning only considers one or quite few entities from the class of analysis.

Cognitive mapping states a topology of the subject of study. This structure is sketched as a digraph of cause-effect relationships between concepts. What is more, a CM pursues to simulate behavior and outcomes through causal reasoning. A CM draws causal conclusions based upon incomplete inductive reasoning. The knowledge induced depicts the variation about the degree of activation of the concept in the context domain, and traces the dynamic behavior of the evolution of the concepts' states along the time. In summary, a CM is a particular point of view about a subject that is qualitatively outlined by concepts linked by causal relations, which is oriented to predict causal behaviors and outcomes.

CM's are exposed by elements and grammatical structures stemmed from natural language. Although, the structure of the thought is the same for the individuals; no matter the place, time, cultural level and language; the same thought owns several forms for being expressed according to the language spoken by the individuals. Wherefore, concepts are expressed by *terms* that identify objects, phenomena, properties and relationships, whilst whole thoughts are pointed out by *sentences*.

Thus cognitive mapping is an internalization-externalization [Leont'ev, 1978] process that represents qualitative knowledge, which is believed to be true according to the particular point of view of the individual. Usually, this kind of knowledge is characterized as: Unilateral, uncertainty, imprecise, unstable, incomplete and not universally sounded. However this type of knowledge is a mirror that reveals how the person thinks and beliefs. Besides it is a source of knowledge for explaining his/her behavior, assumptions and predictions.

2.2 Cognitive Maps Formal Model

Essentially, a CM is stated by equation (1), where the CM is a digraph fixed by concepts (C) and cause-effect relationships (A). Concepts are pictured as nodes, whilst causal relations are traced as arcs. A causal relation ($c_c \rightarrow c_e$) points out that: A *cause* concept (c_i) exerts on an *effect* concept (c_j). Usually the relationship states direction and intensity. Direction represents the nature of the bias, which can be positive, negative or neutral. Whereas, intensity expresses magnitude scales that are given away by qualitative values, crisps values or continuous values. A positive causal relation, as ($c_i \rightarrow + c_j$), means that: c_i excites or enhances c_j. Thus, when c_i is biased positively, c_j will be altered positively too. However, if c_i is influenced negatively c_j will be altered negatively too. Negative relationships, as ($c_k \rightarrow - c_m$), declare that: c_k inversely biases c_m. Wherefore, if c_k is activated negatively, then c_m will be influenced positively. But, if c_k is affected positively, c_m will be altered negatively. Finally, neutral relationships claim that: No matter the direction of the bias on a cause concept c_c, there will not be any bias on an effect concept c_e.

$$CM := (C, A) . \tag{1}$$

Concepts depict qualitative measures about the state that the entities own in a given instant (t_i). The values attached to the concepts' states reveal intensities of activation regarding to variations or levels for the entities after a while. A variation value reveals the magnitude of the change occurred on a concept at the end of a period. A level value points out deviations from the normal state associated to the concept. These kinds of state values never depict the real value of the concept. However they are labeled by intuitive values, qualitative terms, crisp values or continuous values.

The topology of the CM's sets three types of causal relationships: Direct, indirect and feedback. When two concepts ($c_a \rightarrow c_z$) hold a cause-effect relationship, it is said that a *direct relation* involved them, as in Fig. 1[a]. However, if at least one concept, c_b, appears in the middle of the way between $c_a \rightarrow c_b \ldots\rightarrow c_z$, it is acknowledge that an *indirect relationship* links them, in the way sketched in Fig. 1[b]. These kind of indirect relations are based upon the syllogism hypothetic. In regards to causal feedback, most CM's models focus on cyclic flows, where at least two different concepts meet each other, as ($c_a \rightarrow c_b \rightarrow \ldots \rightarrow c_z \rightarrow c_b$). Although, only few versions consider self-feedback as: $c_a \rightarrow c_a$.

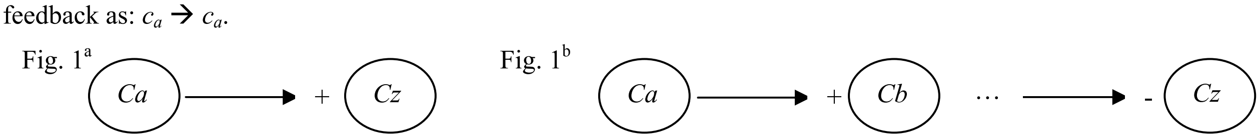


Fig. 1. Causal relationships: [1[a]] Direct relation between *Ca* and *Cz*; [1[b]] Indirect relation between *Ca* and *Cz*

Regarding with causal inferences, an underlying assumption states that given an event $c_a(t_i)$, the event $c_b(t_j)$ can only be a consequence from c_a, if and only if $t_i < t_j$. Moreover, causal inferences estimate the alteration of the concepts' states instead of real values. The mathematical reason is that the equations applied do not include the current state of the concept. In addition, causal inferences outline the accumulative effect that several causes exert on a given one. This kind of effect reveals an additional degree of intensity in the final effect outcome by two or more causes on a given effect concept. Generally, causal inferences are achieved under deterministic and monotonic paradigms.

A simulation process estimates causal behavior and outcomes. Thus, a set of initial values for the concepts' states are estimated and assigned to the concepts of the CM. Afterwards, it begins a cycle that is represented by discrete increments of time that gradually transform the values attached to the concepts' states. During iterations (t_j), causal effects are triggered according to the topology of the CM. As a result, new values for the concepts' states are outcome. These values represent a pattern that corresponds to a point in a search space $m_1 * m_2 * .. * m_n$ dimensional. Where n is the number of concepts, and m_i the number of instances values for the concept i. The simulation breaks down when the process arrives at a stable situation, a pattern of values or a chaotic attractor. This means, that the simulation seeks convergence regions for the concepts. These regions represent stable situations where do not occur any more changes in the pattern. Otherwise, the simulation could meet chaotic attractors, which are regions where it is not possible to find out fixed patterns.

The design of the CM is fulfilled by interviews, documental forms, structured methods and the grid technique. The first two methods identify relevant concepts and causal relationships from the opinions externalized by specialists. In regards structured methods, the modeler shows a relation of concepts to the specialists in the domain of study. So they proceed to choose the most relevant concepts and outline their causal relations. Whereas the grid technique is a semi-structured method oriented to represent an individual's multiple perspectives. The baseline of this technique is the Personal Construct theory, stated by Kelly (1955). Based upon this approach a team of persons define dimensions about their subject of analysis. When the modeler of the CM is a group of individuals, two strategies are considered: Aggregation and global. Aggregation is a bottom-up strategy that integrates the individual's opinions into a whole mental model [Aguilar, 2004]; whilst, the global strategy stems a consensus about an issue that is analyzed from several points of view.

3 Qualitative Cognitive Maps

Qualitative CM's are devoted to depict purely causal relationships, without taking into account values for the concepts' states. This kind of CM's has been applied by Chaib-draa (2002) to deal with subjective views and by Eden (1979) in management sciences. What is more, Klein and Cooper (1982) worked with CM's in game theory problems, whilst Montezemi and Conrath (1986) carried out information analysis through qualitative CM's. Thus, in this section we bring out the underlying formal model for the qualitative CM's. Afterwards we introduce an application of the qualitative CM's for Student Modeling [Peña and Sossa, 2004; Peña 2005].

3.1 Qualitative Cognitive Maps Formal Model

The source of the qualitative CM's is the structure of decision that Axelrod (1976) fulfilled for modeling foreign affairs. Afterwards, the former model was enhanced by Nakamura et al., (1982) through their work for decision support using a causation knowledge base. Finally, Chaib-draa (2002) straightened some pitfalls founded in the former version. Wherefore, in the remainder of this section is summarized these contributions into a holistic formal model.

The basic model for the CM's labels causal relationships by the set ρ. It owns eight values ρ ={+, -. 0, ⊕, Θ, ±, a, ?}, whose instances are explained in the Table 1. The manipulation of the set ρ of causal relations is achieved through four operators: Union (∪), intersection (∩), multiplication (*), and addition (|). The laws of union and intersection, stated in the set of equations (2), are obtained by considering +, -. 0, ⊕, Θ, ±, ?, and a as shorthands for the sets {+}, {-}, {0}, {+, 0}, {-, 0}, {+, -}, {+, 0, -} and {} respectively.

$$(2^a)\ \oplus = 0 \cup +;\ (2^b)\ \Theta = 0 \cup -;\ (2^c)\ \pm = + \cup -;\ (2^d)\ ? = 0 \cup + \cup -;\ (2^e)\ a = + \cap 0 = + \cap - = 0 \cap -;\ (2^f)\ \forall x : a \cup x = x\ . \quad (2)$$

The indirect relations are estimated through six laws of the multiplication {1[a] to 1[f]} set in Table 2[a]; whilst direct relations are stemmed from the six addition laws {2[a] to 2[f]} depicted in Table 3[a]. For instance, the causal chain (c_a →- c_b →- c_c) outcomes a final effect +, due to law {1[d]}. When a given concept c_z is affected by two or more cause concepts, through their respective direct relationships, an accumulated total bias is outcome by the laws of sum, e.g., two paths (c_a →- c_z) (c_b →+ c_z) produces the final effect *?* according to the law{2[d]}.

Regarding the distributive laws {1[e]} and {2[e]}, these laws represent the use of the distributive property for any instance related with ∪. For instance, - | Θ = - | 0 ∪ - = - | 0 ∪ - | - = - ∪ - = -. This equivalence depicts the value of the entry (-, Θ) in the Table 3[b]. This outcome is grown from the successive application of the equation (2[b]) and the laws {2[e]}, {2[a]} and {2[c]}.

Table 1. Description of the labels for causal relationships of the qualitative CM's

Label	Meaning
C_a → + C_b	C_a *excites* C_b. So an increment in C_a promotes C_b. Also, a decrement in C_a inhibits C_b
C_a → - C_b	C_a *hurts* C_b. So an increment in C_a inhibits C_b. Also, a decrement in C_a promotes C_b
C_a → 0 C_b	C_a *is neutral to* C_b. So an increment or decrement in C_a has no effect on C_b
C_a → ⊕ C_b	C_a *excites or is neutral to* C_b. So C_a does not inhibits C_b
C_a → Θ C_b	C_a *hurts or is neutral to* C_b. So C_a does not excites C_b
C_a → ± C_b	C_a *excites or hurts to* C_b. So C_a is not neutral to C_b
C_a → ? C_b	C_a *excites or hurts or is neutral to* C_b. So C_a has some effect on C_b
C_a → *a* C_b	C_a *does not excite, neither hurts nor is neutral to* C_b. So C_a has an ambivalent relation with C_b

Table 2. Laws for the multiplication *: [2[a]] Specific Laws; [2[b]] Table stemmed from the laws of multiplication *

2[a]. Specific Laws of multiplication *	2[b]	?	±	Θ	⊕	-	+	0	*a*
For any x, y Є C	?	?	?	?	?	?	?	0	
{1[a]} $+ * y = y$	±	?	±	?	?	±	±	0	*a*
{1[b]} $0 * y = 0$	Θ	?	?	⊕	Θ	⊕	Θ	0	
{1[c]} $a * y = a$, if $y \neq 0$	⊕	?	?	Θ	⊕	Θ	⊕	0	
{1[d]} - * - = +	-	?	±	⊕	Θ	+	-	0	*a*
{1[e]} * do ∪ (distributive law)	+	?	±	Θ	⊕	-	+	0	*a*
{1[f]} $x * y = y * x$	0	0	0	0	0	0	0	0	0
	a		*a*			*a*	*a*	0	*a*

Table 3. Laws for the addition |: [3[a]] Specific Laws; [3[b]] Table stemmed from the laws of addition |

3[a]. Specific Laws of addition |	3[b]	?	±	Θ	⊕	-	+	0	*a*
For any x, y Є C	?	?	?	?	?	?	?	?	*a*
{2[a]} $0 \mid y = y$	±	?	±	?	?	?	?	±	*a*
{2[b]} $a \mid y = a$	Θ	?	?	Θ	?	-	?	Θ	*a*
{2[c]} $y \mid y = y$	⊕	?	?	?	⊕	?	+	⊕	*a*
{2[d]} + | - = ?	-	?	?	-	?	-	?	-	*a*
{2[e]} | do ∪ (distributive law)	+	?	?	?	+	?	+	+	*a*
{2[f]} $x \mid y = y \mid x$	0	?	±	Θ	⊕	-	+	0	*a*
	a	*a*	*a*	*a*	*a*	*a*	*a*	*a*	*a*

However, this formal model is not consistent for some instances of the laws of multiplication, due to few of them lead to contradictory results, whose entries in Table 2[b] have no value. For instance, based upon the law {1[c]} *a* * Θ= *a*. But, according to the successive application of the equation (2[b]), laws {1[e]}, {1[b]}, {1[c]}, and equation (2[f]), the outcome is *0*, as follows: *a* * Θ = *a* * (0 ∪ -) = *a* * 0 ∪ *a* * - = 0 ∪ *a* = 0. This kind of drawback is straightened by

Chaib (2002) through his model based upon relational algebra. What is more, Chaib states a semantic and enhances the sound of the formal model.

The labels attached to the causal relationships are stored into an adjacency matrix (A), like the one stated in Table 5[1]. Matrix A owns a dimension of $n * n$, where n is the number of concepts and the entry $A_{i,k}$ depicts the value of the link that goes from c_i to c_k. Based on the transformation of the matrix A, qualitative CM's achieve the causal behavior and outcomes.

Due to operators | and * are lifted to matrices, CM's use the classic definitions for matrix addition, multiplication, and n^{th} power, which are stated in ($3^{a,b,c}$) respectively. So the total effect of one concept c_a on another c_z is achieved by the sum of the indirect effects of all paths stemmed form c_a to c_z. Wherefore, the total effect matrix A_t owns as it's a, z^{th} entry the total effect of the concept c_a on the concept c_z at step t, based on (4^a). Because the addition operator | is $\subseteq$ monotonic, there is a k such that it represents the number of arcs in the longest path between c_c and c_e concepts, as is pointed out in (4^b).

$$(3^a)\ (A \mid B)_{ce} = A_{ce} \mid B_{ce}; \quad (3^b)\ (A * B)_{ce} = (A_{c1} * B_{1e}) \mid \cdots \mid (A_{cn} * B_{ne}); \quad (3^c)\ A^1 := A;\ and\ \ A^n := A * A^{n-1}. \tag{3}$$

$$(4^a)\quad A_t := A^1 \mid A^2 \mid A^3 \mid \cdots\cdots; \qquad (4^b)\quad A_t := A^1 \mid A^2 \mid A^3 \mid \cdots\cdots \mid A^k. \tag{4}$$

3.2 Case of Study

As a tool for modeling causal phenomena from a pure qualitative perspective, in this section we introduce our CM's approach for a Web-based Education System (WBES) [Peña 2005]. First of all, teaching-learning experiences delivered by a WBES can be studied as cause-effect events. Where, teaching corresponds to the cause and learning to the effect. In addition, qualitative CM's can be used to model the planning of the sequencing of teaching experiences, the organization of the content, and the anticipation of the learning effects. Thus, in this section, we focus on the Student Modeling as the process devoted to point out a mental model of the individual and the knowledge about the domain of study.

The approach for Student Modeling is based on a Multi-Agents System (MAS). These kinds of agents play a particular role for revealing the perspective of a given actor such as: Student, system, coach or peer. As a result, the point of view about a specific issue is different between the involved actors. Therefore, it is necessary to: Deal with multiple perspectives, pursue consensus, model a common view and simulate causal outcomes. Wherefore, as specified by the object oriented design (OO), the first task is fulfilled by agents that express the beliefs of the student and his/her coach about the teaching topic.. As a result, two CM's are outcome; the first one corresponds to the student's agent in Fig. 2^a, and second one to the coach's agent in Fig. 2^b.

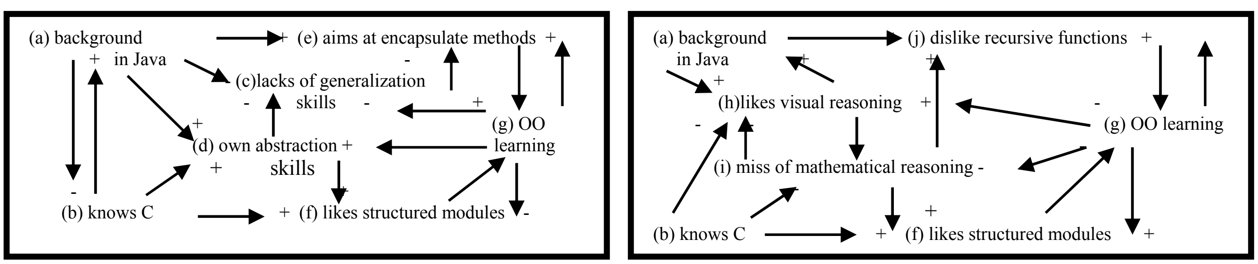


Fig. 2. Qualitative CM's: [2^a] Student's agent point of view; [2^b] Coach's agent beliefs

The CM's brought out by the student's agent, sketched in Fig. 2^a, states seven concepts, labeled by letters (a) to (g). While, the CM generated by the coach's agent depicts seven concepts too. However, only four concepts - (a), (b), (f), (g) - are common to both point of views. In regards with causal relationships, there are several coincidences as the corresponding to the relations between $f \rightarrow -g$, and $g \rightarrow - f$. But, according to particular beliefs there are causal relations in one CM that are not included in the other one, e.g., in Fig. 2[a] there are recursive relationships between concepts (*a*), (*b*) that are not taken into account by the coach's agent in Fig. 2[b].

The analysis of the opinions of both agents is done through the adjacency matrix A sketched in Table 4, where, rows correspond to the cause concepts and columns represent effect concepts. Entries, as $A_{c,e}$, with no value mean that both views agree that there is none causal relationship from the cause concept c to the effect one e. Entries with only one value reveal *partial perspectives*. When only the left value is instantiated, e.g., $A_{a,b}$ = +, it means that student's agent owns a belief that its coach's pair ignores. In the opposite side, entries with only the right value, e.g., $A_{b,h}$ = ,-; identifies partial opinions stemmed from coach's agent, which are not know by student's agent. Finally, entries with a couple of values reveal the perspective of both agents regarding the belief that the cause concept exerts on effect concept. When both values are labeled by the same causal sign, e.g., $A_{b,f}$ = +,+; is acknowledge as a *coincidence*, otherwise is stated as a *disagreement*, as $A_{f,h}$ = -,+.

In order to achieve a consensus from multiples perspectives, we can use mechanisms such that: Negotiation, mediation and auctions. A negotiation schema underlines the relevance of some concepts and causal relationships with the purpose that actor's agents reach an agreement. A mediation approach tries to surround consensus and particular point of views among the agents [Chaib-draa, 2002]. Auctions, involve deliberative processes under changing conditions constrained by uncertainty and time [Noriega, 1997]. Thus, the application of these kinds of strategies yields holistic perspectives, as the one sketched in Fig. 3. In such figure, appears a qualitative CM that mixes the views from Student's and Coach's agents.

Table 4. Multiples views adjacency matrix

4	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**
a		-,	-,	+,	+,			,+		,+
b	+,			+,		+,+		,-	,-	
c					-,					
d			-,			+,				
e							+,			
f							-,+			
g			-,	+,	+,	-,-		,+	,-	,+
h	,+								,-	
i						,+		,-		
j							,-			

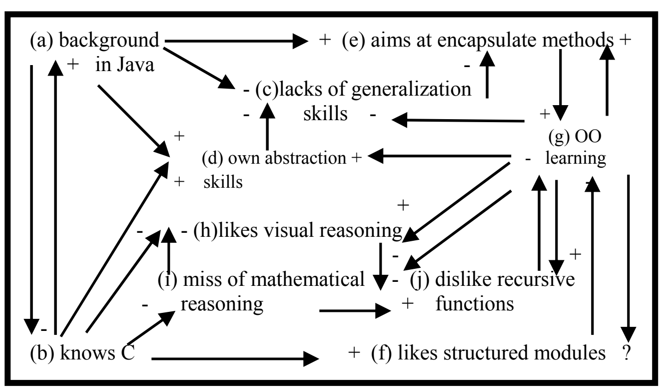


Fig. 3. Qualitative CM with a holistic view from Student's and Coach's agents

The simulation of the causal behavior is done by successive transformations over the adjacency matrix A, as follows: Firstly, the *initial* adjacency matrix A^1, stated in Table 5[1], is stemmed from the values attached to the causal relationships of the CM. Afterwards, based upon (3[c]), the process takes up the estimation of *indirect* causal effects achieving the *second* adjacency matrix A^2, which is illustrated in Table 5[2]. The simulation carries out *consecutive* adjacency matrices $A^{3..t}$ that are depicted in Tables 5[3] to 5[7]. In this case, the simulation breaks down when it meets a pattern of values, as the one identified in Table 5[8]. Finally, the total effect between each couple of concepts is the sum of the indirect affects of all paths that join them according with the *total effect* adjacency matrix A_t brought about by the set of equations (4).

Table 5. Set of adjacency matrices: [5[1]] Initial adjacency matrix A^1; [5[2]] Second adjacency matrix A^2, after step 1

5[1]	a	b	c	d	e	f	g	h	i	j
a		-	-	+	+					
b	+			+		+		-	-	
c					-					
d			-							
e							+			
f							-			
g			-	+	+	?		+	-	
h									-	
i								-		+
j							-			

5[2]	a	b	c	d	e	f	g	h	i	j
a	-		-	-	+	-	+	+	+	
b		-	-	+	+		-	+	+	-
c							-			
d					+					
e			-	+	+	?		+	-	
f			+	-	-	?		-	+	
g			-		+		?	+	-	-
h								+		-
i							-		+	
j			+	-	-	?		-	+	

Table 5. (Continuation) set of adjacency matrices: Tables 5[3] to 5[8], correspond to adjacency matrices $A^3..A^8$ after steps 2 to 7 respectively

5[3]	a	b	c	d	e	f	g	h	i	j
a		+	?	?	?	?	+	?	-	+
b	-		?	-	?	?	+	?	?	+
c			+	-	-	?		-	+	
d							+			
e			-		+		?	+	-	-
f			+		-		?	-	+	+
g			?	?	?	?	+	?	?	-
h							+		-	
i			+	-	-	?		-	+	+
j			+		-		?	-	+	+

5[4]	a	b	c	d	e	f	g	h	i	j
a	+		?	+	?	?	?	?	?	-
b		+	?	?	?	?	?	?	?	?
c			+		-		?	-	+	+
d			-	+	+	?		+	-	
e			?	?	?	?	+	?	?	-
f			?	?	?	?	-	?	?	+
g			?	+	?	?	?	?	?	?
h			-	+	+	?		+	-	-
i			+		-		?	-	+	+
j			?	?	?	?	-	?	?	+

5[5]	a	b	c	d	e	f	g	h	i	j
a		-	?	?	?	?	?	?	?	?
b	+		?	?	?	?	?	?	?	?
c			?	?	?	?	-	?	?	+
d			-		+		?	+	-	-
e			?	+	?	?	?	?	?	?
f			?	-	?	?	?	?	?	?
g			?	?	?	?	?	?	?	?
h			-		+		?	+	-	-
i			?	?	?	?	-	?	?	+
j			?	-	?	?	?	?	?	?

5[6]	a	b	c	d	e	f	g	h	i	j
a	-		?	?	?	?	?	?	?	?
b		-	?	?	?	?	?	?	?	?
c			?	-	?	?	?	?	?	?
d			?	?	?	?	+	?	?	-
e			?	?	?	?	?	?	?	?
f			?	?	?	?	?	?	?	?
g			?	?	?	?	?	?	?	?
h			?	?	?	?	+	?	?	-
i			?	-	?	?	?	?	?	?
j			?	?	?	?	?	?	?	?

5[7]	a	b	c	d	e	f	g	h	i	j
a		+	?	?	?	?	?	?	?	?
b	-		?	?	?	?	?	?	?	?
c			?	?	?	?	?	?	?	?
d			?	+	?	?	?	?	?	?
e			?	?	?	?	?	?	?	?
f			?	?	?	?	?	?	?	?
g			?	?	?	?	?	?	?	?
h			?	+	?	?	?	?	?	?
i			?	?	?	?	?	?	?	?
j			?	?	?	?	?	?	?	?

5[8]	a	b	c	d	e	f	g	h	i	j
a	+		?	?	?	?	?	?	?	?
b		+	?	?	?	?	?	?	?	?
c			?	?	?	?	?	?	?	?
d			?	?	?	?	?	?	?	?
e			?	?	?	?	?	?	?	?
f			?	?	?	?	?	?	?	?
g			?	?	?	?	?	?	?	?
h			?	?	?	?	?	?	?	?
i			?	?	?	?	?	?	?	?
j			?	?	?	?	?	?	?	?

The process for achieving the early adjacency matrices $A^{1..8}$ consist of the multiplication of matrices stated in (3[c]). Thus, for each cycle the current causal effect n is estimated as: $A^n = A^1 * A^{n-1}$. The outcome is stemmed from equations (3[a]) and (3[b]), and by the laws {1[a..f]} and {2[a..f]}. These computations are applied over A^n as follows:

In Table 5[1], A^1 points out the *direct* causal relations between each pair of concepts, e.g., entry $A_{a,c}$= - corresponds to the arc $c_a \rightarrow - c_c$ of the CM. Also, couples of concepts with neutral relations, as $c_a \rightarrow 0\ c_f$, own entries with no value, as $A_{a,f}$ = .

In regards the second adjacency matrix A^2, sketched in Table 5[b], it is the result of the shortest *indirect* causal effect between a pair of concepts. This means that the length of the path between the involved concepts includes just one intermediary concept. For instance, although concepts c_b and c_e are joined by two causal paths $c_b \rightarrow + c_a \rightarrow + c_e$ and $c_b \rightarrow + c_d \rightarrow - c_c \rightarrow - c_e$, the entry $A^2_{b,e}$ = + gives away only the outcome of the first path according with {1[a]}. What is more, when there are more than one path with the same length, their causal effects are added into the respective entry, e.g., concepts c_b and c_c are linked by two paths $c_b \rightarrow + c_a \rightarrow - c_c$ and $c_b \rightarrow + c_d \rightarrow - c_c$. So entry $A^2_{b,c}$ becomes = - | - = + based on {1[a], 1[a], 2[c]}.

Regarding the third to the seventh adjacency matrices A^3 to A^7, these matrices contain the total causal effect corresponding to the respective length of the path(s) that join each pair of concepts. So in $A^3_{h,g}$ = + corresponds to the causal path $c_h \rightarrow - c_i \rightarrow + c_j \rightarrow - c_g$, due to {1[a], 1[d]}. Moreover, the value of the entry $A^4_{h,d}$ = +, corresponds to the effect of the path $c_h \rightarrow - c_i \rightarrow + c_j \rightarrow - c_g \rightarrow + c_d$, because of {1[a], 1[d], 1[a]}. However, in the step 5 the total effect for these concepts is none, or neutral, $A^5_{h,d}$ = , why? There is no path of length 5 between concepts c_h and c_d. But, one step ahead the outcome is $A^6_{h,d}$ = ?, what happened? We can see that there are three paths of length 6 with the following description: $c_h \rightarrow - c_i \rightarrow + c_j \rightarrow - c_g \rightarrow + c_e \rightarrow + c_g \rightarrow + c_d$ = +; $c_h \rightarrow - c_i \rightarrow - c_h \rightarrow - c_i \rightarrow + c_j \rightarrow - c_g \rightarrow + c_d$ = +; $c_h \rightarrow - c_i \rightarrow + c_j \rightarrow - c_g \rightarrow ?\, c_f \rightarrow - c_g \rightarrow + c_d$ = ? Thus, the sum of the three outcomes is + | + | ? = ?, since {2[c], entry +, ? of Table 3}. One step further, the entry $A^7_{h,d}$ = +, due to there is only one path of length 7, such that: $c_h \rightarrow - c_i \rightarrow + c_j \rightarrow - c_g \rightarrow - c_c \rightarrow - c_e \rightarrow + c_g \rightarrow + c_d$ = +, owing to {1[a], 1[d], 1[a],1[d], 1[a], 1[a]}.

Finally, A^8 presents the first matrix of a set of four matrices that conforms a pattern of values of a convergence region. So the difference among the four matrices corresponds to four entries: $A_{a,a}$, $A_{a,b}$, $A_{b,a}$, and $A_{b,b}$, whose respective values for matrices A^8, A^9, A^{10}, and A^{11}, are: {+, 0, -, 0}, {0, -, 0, +}, {0, +, 0, -}, and {+, 0, -, 0}. The rest of the entries for each matrix is ? This means that since the eighth iteration, paths with this length or larger give away the same causal effects.

4 Fuzzy Cognitive Maps

Many times, the cognitive mapping is constrained to deal with uncertainty in order to model real issues with more accuracy. So in this section we introduce the fuzzy version for the CM's, which were stated by Kosko (1986). According with this approach, the causal relationships bring out different gray levels of intensity. Moreover, it is possible to take over the changes on the state of the concepts during the simulation process. The background of the fuzzy CM's comes from fuzzy logic and neural networks. The diversity of the fuzzy CM's applications includes: Cognitive ergonomy [Parenthoën, et al., 2002], managing synergies [Koulouriotis et al., 2003], and Virtual reality [Mohr, 1997]. Whereby, we get down setting the formal model and afterwards we carry on with the former case of study since its fuzzy point of view [Peña et al., 2005[a]].

4.1 Cognitive Maps Former Model

Basically the fuzzy causal modeling is achieved from two perspectives: Qualitative and quantitative. Firstly, the designer depicts concepts and relations as linguistic variables. Where, concepts outline the change of the state or the deviation from its normal level of the entity that they represent, and causal relationships give away the tendency and magnitude of the bias. Later on, a universe of domain is attached to each concept and to the whole set of causal relations in order to state the set of instance values. Thus, concepts are instantiated with linguistic values such that: Increase, decrease, low, very high and so on. Whereas causal relationships are measured by fuzzy values, as: Positive strong, negative weak, ignores, etc. Next, a fuzzyfication function is set to convert qualitative terms to quantitative values, which can be crisp or continue. Usually, crisp values are bivalent or trivalent, as {0, 1} or {-1, 0, 1}, and continues values fall into a range as [0, 1] or [-1, 1]. Finally, a threshold function is applied to the concept's state value in order to normalize it into the corresponding set or range.

The data structure used for the fuzzy CM's includes a state vector (*S*) and an adjacency matrix (*A*). The vector *S* owns a dimension of *n* that corresponds to the number of concepts. Vector *S* holds the initial state values assigned to the concepts, as a result of the fuzzyfication. During the simulation, new vectors *S(t)* are outcome as a consequence of the iterations. Wherefore, the causal behavior is stemmed from the variations recorded into the several versions of vector *S*. In regards with the adjacency matrix *A*, this matrix has a dimension of $n*n$ for storing the value of the relationships. However, *A* remains static through the cycle of causal inferences, in contrast with qualitative CM's.

The simulation process is accomplished during discrete time series, when iteratively the inference engine applies the summation and threshold processes to the state vector *S(t)*. The state of a given concept $s_i(t)$ is obtained from the prior states of all causal concepts $s_k(t-1)$. These concept's states $s_k(t-1)$ are multiplied by the value of the causal relationship a_{ik}, where the antecedent concept c_k bias to the consequent concept c_i. The sum of these products $r_i(t)$ is a rough value that is normalized by a threshold function *u*. This function constrains the new state values *S(t)* to a set or a range of permissible values according with the type of value attached to the concept. The application of *u* gives up the possibility of quantitative results, but it supports the comparison between concept's states.

Whereby prior to get down to the simulation, the initial state vector *S(1)* is set with the outcome of the fuzzification of the qualitative terms. Moreover, the adjacency matrix *A* is fixed with the quantified relation values. Afterwards, the fuzzy causal engine takes over the simulation through successive increments of time *t=1, t=2,..* At each step (*t*), the current version for the vector *S(t)* is estimated through equation (5^a), where the entry $s_i(t)$ is the new state value for the concept c_i that the threshold function *u* yields from the rough state value $r_i(t)$. The value $r_i(t)$ is fulfilled according with (5^b). This rough value brings out the accumulated influence that antecedent concepts c_k put on consequent concept c_i. The causal bias grows from the product between the value of the causal relationship a_{ik} and the former value of the antecedent concept $s_k(t-1)$. Afterwards, $r_i(t)$ is normalized by one of the following threshold functions *u*: (6^a) achieves crisps bivalent values, (6^b) carries out crisps trivalent values, (6^c) puts in continues values into the range [0, 1], and (6^d) yields continues values into the range [-1, 1]. Equations (6^c) and (6^d) set up logistic and tangent signals with *c* =5 as a degree of fuzzification [Mohr, 1997].

$$(5^a) \quad s_i(t) = u(r_i); \qquad (5^b) \quad r_i = \left(\sum\nolimits_{k=1}^{n} w_{ik} * s_k(t-1) \right). \tag{5}$$

$$(6^a) \; u(r_i) = 0, r_i \le 0; \quad u(r_i) = 1, r_i > 0; \quad (6^b) \quad u(r_i) = -1, r_i \le -0.5; \; u(r_i) = 0, r_i > -0.5 \wedge r_i < 0.5; \; u(r_i) = 1, r_i \ge 0.5. \tag{6}$$
$$(6^c) \quad u = 1 \;/\; (1 + e^{-cr}); \qquad (6^d) \quad u = (e^{cr} - e^{-cr}) \;/\; (e^{cr} + e^{-cr}).$$

The equilibrium state of a fuzzy CM is met when one or more state vectors *S(m..)* occur repetitively since a given iteration *m*. If this situation happens, the process has reached an equilibrium state and the simulation is broken down. Thus, a Fuzzy CM with crisp values own a high chance to reach an equilibrium state, due to their threshold functions force rough values to discrete ones. Whereas, a fuzzy CM that uses continues values may become nonlinear under complex feedback dynamics, unless it lacks of feedbacks. If such condition is held, the stability is checked in terms of the eigenvalues of the adjacency matrix. Therefore, if all the eigenvalues have negative real parts, the CM achieves some form of stability.

4.2 Case of Study

Based on the application introduced in section 3.2, now the qualitative CM for the holistic Student Model is turned into its fuzzy version. Wherefore, it is necessary to depict the sign values of the causal relations to fuzzy terms and afterwards to crisp or continue values. Moreover, the concepts are measured by linguistic values, that later on they are stated as real or integer values. These tasks bring out a fuzzy CM, its corresponding adjacency matrix and four versions of the initial state vector. Thus, in Fig. 4 is shown the fuzzy CM, whilst in Table 6 appears its adjacency matrix *A*. Also, in the Table 7 appears four series of state vectors with bipolar, trivalent, and real values into the ranges [0, 1] and [-1, 1]. These series are organized in four tables, Table $7^{1..4}$. At each table 7^i, the columns correspond to the concepts state values, whilst rows to the iteration carried out during the simulation. This means, that a given Table7[i] depicts the outcomes achieved by a specific scenario of study according to the type of the value. Thus, the first row represents the initial state values estimated for the concepts and the following rows to the successive states values which were fulfilled by the set of equations (5) and (6).

Table 6. Fuzzy adjacency matrix for the Fuzzy CM in Fig. 4

A	a	b	c	d	e	f	g	h	i	j
a		-.2	-1	0.6	0.9					
b	0.5			0.4		0.3		-.1	-.7	
c					-.9					
d			-.7							
e							0.5			
f							-1			
g			-.3	0.2	0.7	0.1		0.4	-.8	0.4
h									-.4	
i								-.7		0.9
j							-.6			

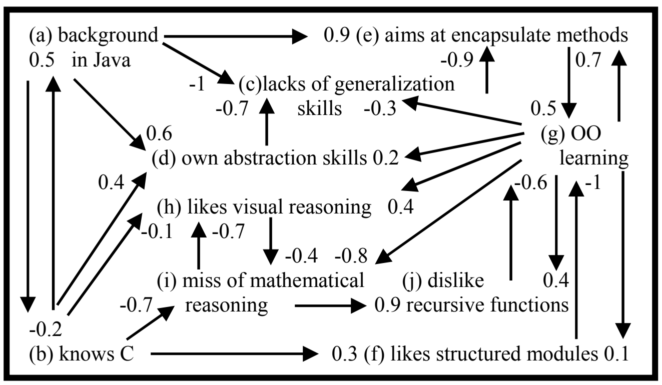


Fig. 4. Fuzzy CM with a holistic view from Student's and Coach's agents

Table 7. Set of vector states: [7^1] Bivalent; [7^2] Trivalent; [7^3] Logistic signal for [0, 1], [7^4] Tangent signal for [-1, 1]

7^1	a	b	c	d	e	f	g	h	i	j
0	0	1	1	0	0	1	0	1	1	1
1	1	0	0	1	0	1	0	0	0	1
2	0	0	0	1	1	0	0	0	0	0
3	0	0	0	0	0	0	1	0	0	0
4	0	0	0	1	1	1	0	1	0	1
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0

7^2	a	b	c	d	e	f	g	h	i	j
0	-1	1	1	-1	-1	1	-1	1	1	1
1	1	0	1	0	-1	0	-1	-1	0	1
2	0	0	-1	0	-1	0	-1	0	1	0
3	0	0	0	0	0	0	-1	-1	1	1
4	0	0	0	0	-1	0	-1	-1	1	1
5	0	0	0	0	-1	0	-1	-1	1	1

7^3	a	b	c	d	e	f	g	h	i	j
0	0	1	1	0	0	1	0	1	1	1
1	.92	.50	.50	.88	.01	.82	0.0	.02	0.0	.99
2	.78	.28	0.0	.98	.87	.68	0.0	.44	.14	.50
3	.67	.31	0.0	.95	.97	.60	.06	.35	.13	.65
4	.68	.34	0.0	.94	.96	.62	.07	.38	.12	.67
5	.70	.34	0.0	.94	.96	.63	.06	.39	.10	.66
6	.70	.33	0.0	.94	.97	.63	.06	.40	.10	.64
7	.70	.33	0.0	.94	.97	.63	.07	.40	.10	.64
8	.70	.33	0.0	.94	.97	.63	.07	.41	.10	.64
9	.70	.33	0.0	.94	.97	.63	.07	.41	.09	.64
10	.70	.33	0.0	.94	.97	.63	.07	.42	.09	.63
11	.70	.33	0.0	.94	.97	.63	.07	.42	.09	.63

7^4	a	b	c	d	e	f	g	h	i	j
0	-1	1	1	-1	-1	1	-1	1	1	1
1	.99	.76	1	-.96	-1	.76	-1	-1	-.91	.99
2	.96	.76	-.09	1	-1	.56	-1	.67	1	-1
3	-.96	-.74	-1	.35	.84	-.93	-.98	-1	1	.99
4	-.95	.74	1	-1	-1	-.92	1	-1	1	.99
5	.95	.74	1	-.35	-1	.95	-.7	-.95	-1	1
6	.95	-.74	-.99	1	-.99	.64	-1	.94	.97	-1
7	-.95	-.74	-1	.35	1	-.92	-.99	-1	1	.98
8	-.95	.74	1	-1	-1	-.92	1	-1	1	.99
9	.95	.74	1	-.35	-1	.95	-.7	-.95	-1	1
10	.95	-.74	-.99	1	-.99	.64	-1	.94	.97	-1
11	-.95	-.74	-1	.35	1	-.92	-.99	-1	1	.98

In Table 7[1] we analyse the evolution of the CM through bivalent values, where 0 means that the concept's state is not perturbed in that moment – inactive -, and 1 reveals that the concept's state is altered in the current time –active-. The simulation estimates the behavior of the concepts during five steps, because the process converges since the fifth iteration. So the variations occurred on the concepts' states can be stemmed as follows: Concepts *b*, *c*, and *i* arrive to their final state after just one step. But, concepts *d*, *e*, *f*, *h*, and *j* carry on with changing until the fifth step. Along the process, these concepts meet ups and downs, e.g., concept *f* brings about active states, after inactive, next active and finally inactive.

Semantically speaking the CM brings out predictions such that: Although concept *a*, background in Java, is primary stimulated, after that it is not more biased. Regarding concepts *b*, *c*, and *i*, they initially revel that the student is familiarized with C, but he/she lacks of generalization skills and he/she miss of mathematical reasoning. However, these factors remain inactive during the simulation, why? Because of these concepts are not enough reinforced by their incoming stimulus. As a consequence, they became irrelevant for the learning of Object Oriented paradigm. In regards with concept *g*, this goal is achieved after three cycles, due to the input influences, stemmed from concepts *a* and *b*, it wastes three steps for biasing the learning of OO. Finally, as a consequence of the feedback outcome by the objective, concepts *d*, *e*, *f*, *h* and *j* are strengthened. Thus, the abstraction skills, the aim at recursive functions, the attraction for structured modules, the visual reasoning and the dislike for recursive functions are encouraged. After that, all the concepts remain inactive.

Regarding with trivalent scenario, the values 1, 0, -1 correspond respectively to: Positive active, indifferent and negative activation states. The simulation converges at the fourth cycle, as it is shown in Table 7[2]. The results give away several behavior patterns. For instance, at the beginning concepts as *b* and *f* own positive activation. Next they change their state to indifferent and during the process they remain with such value. This pattern brings out that the student knows C language and he/she likes structured programming, but these factors are not reinforced during the process. Also, the whole fuzzy CM brings out that the goal, related to learn OO, is not enough stimulated, wherefore it stays with a negative activation. In addition, concepts *e*, *i*, and *j* are turn back to their original state after to bear with a change of state. This means that student maintains his/her former dislike for encapsulate methods and recursive functions. What is more, the simulation reveals that he/she has not improved his/her mathematical reasoning.

In Table 7[3], it is given away the study based on the logistic signal that yields real values into the range [0, 1], where nearby values to 1 depict active states and neighbouring numbers to 0 mean that the concept is inactive. At step 10, the simulation converges into a stable vector. The results bring out predictions such that: The learning OO will be fulfilled with a minimal achievement, whilst the lack of generalization skills and mathematical reasoning will quickly loss relevance. Nevertheless, the student's abstraction skills will be stimulated very high.

Finally, the analysis of the CM based upon tangent signals brings about real values into the range [-1, 1]. Close values to -1 depict negative activations, quantities around 0 reveal indifference, and numbers near to 1 claim positive activations. The simulation breaks down at cycle number 4. Since them, a pattern of four state vectors is repeated with concepts' values that ups and downs alternatively. This result is alike the four pattern convergence region met by the qualitative CM, whose most of its causal influences are *?*. Thus, what is the reason of this phenomenon? According with the topology of the CM, there are two *input* concepts *a* and *b*. None of the other concepts of the CM bias to c_a nor c_b. However, these input concepts own direct and indirect relationships with the remaining concepts. As a consequence, concepts *a* and *b* affect the behavior of the whole CM. Indeed, c_a and c_b hold a deviation-countering loop. This kind of recursive relation, such as $c_a \rightarrow- c_b \rightarrow+ c_a = -$, or $c_b \rightarrow+ c_a \rightarrow- c_b = -$, leads to stabilize the whole fuzzy CM through series of increments followed by decrements. According with Chaib-draa (2002), in a wholistic approach, the whole constrains the concepts and the relations. Thus, it is possible to watch this kind of pattern along the set of four state vectors stated in the group of rows 4-7, 8-11 of the Table 7[4].

5 Rule-Base Fuzzy Cognitive Maps

As a result of the PhD dissertation achieved by Carvalho (2001), the classical fuzzy CM's were enhanced to Rule Base Fuzzy Cognitive Maps (RB-FCM's). This kind of CM meets causal relationships and causal effects with fuzzy logic. Also, RB-FCM's brings up fuzzy causal relations (FCR's) and a fuzzy carry accumulation (FCA). FCR's define causal relationships through fuzzy rule bases, whilst a FCA estimates the *accumulative* effect that an antecedent concept yields on a consequent concept. Shortly, RB-FCM's join the fuzzy logic with the qualitative and fuzzy CM's by means of the formal model introduced next. Moreover, this approach is applied to the Student Modeling, early introduced [Peña et al., 2006].

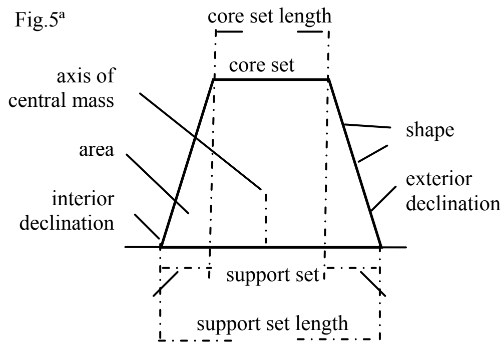
5.1 Formal Model

First of all, RB-FCM's work into a couple of inference lines: Causal and fuzzy. Due to causality is seen as a *variation* upon the state of a given concept; whilst fuzzy line is met as an influence that imposes a *level* on the state of a concept. This perspective brings out the statement of two kinds of concepts: Variations and levels. Whereby, RB-FCM's deal with two kinds of relationships: Causal and influence. A FCR sets up a causal relationship between just one antecedent concept and only one consequent variation concept, whereas, a fuzzy influence relationship (FIR) estimates the classical fuzzy effect that one or more antecedent concepts exert on just one consequent concept. The type of effect brought out by FCR's and FIR's is *accumulative* and *aggregative* respectively. An accumulative effect gives away the displacement of the consequent linguistic terms along the universe of discourse (UoD) based upon their membership degree. Meanwhile, an aggregative effect just adds the membership degrees of the involved consequent linguistic terms without any shifting over the UoD.

Concepts are stated as linguistic variables that are instantiated by linguistic terms. These fuzzy values are grown from the UoD attached to the concept. According with the nature of the concepts, the instances of their respective UoD identify *variations* or *levels* of the state's value. A variation reveals the direction and intensity of the change on the state's value of the entity after a while, i.e., variation values for the *inflation* concept are: *Increases much* and *decrease few*. A level concept claims the absolute state's value of the entity in a given time, i.e., the level values for the *taxes* concept: *Low* and *high*.

Linguistic terms are sketched as fuzzy sets by membership functions. Fuzzy sets own physical properties as support set, area, and many more that are pictured in Fig. 5[a]. Based upon the qualitative intensity or fuzzy level stated by the fuzzy terms, their corresponding fuzzy sets are allocated along the UoD's x-axis. The identification of the point in the x-axis of the UoD where a given fuzzy set is allocated, it is done by a mapping between its linguistic term and a normalized value. This kind of value gives away a *degree of incertitude*. The normalized value is usually represented by a discrete number into the range [-1, 1], which corresponds to the scale of UoD's x-axis. For instance, in Fig. 5[b] some variation fuzzy terms are illustrated along the UoD of a concept as follows: At the left side are: *Decrease much*, *medium* and *low*. As central point is the value *maintains*. Finally, at the right side appear: Increase low, medium and much.

Moreover, all the fuzzy sets *S* associated to the fuzzy values *x* in the UoD *X* attached to a linguistic variable own a membership function μ. This function yields a membership degree given by: $\mu_F(x)\text{:}X \rightarrow [0, 1],\ x \in X$. This value corresponds to the scale for the y-axis of the UoD. Furthermore, neighboring fuzzy sets are considered complementary. So, their point of intersection *x* in the UoD's x-axis owns as a membership degree $y = 0.5$ for each of them. Thus, for any point *x* a maximum of two fuzzy sets corresponds to it. As a result, any point *x* into the UoD's x-axis owns 1 in the y-axis as the membership degree outcome by the sum of its corresponding fuzzy sets. Thus, as much is the linguistic variation stated by the fuzzy set, larger will be the length of its support set and its area. These kinds of constraints are illustrated in Fig. 5[b].



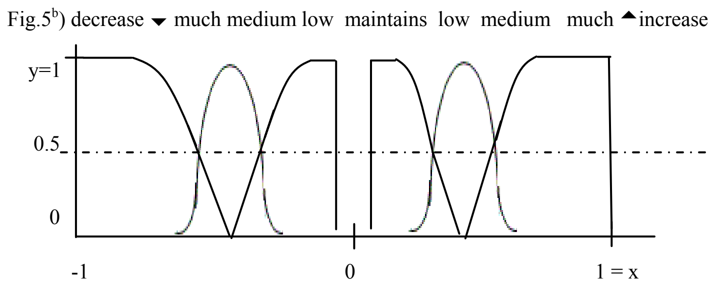


Fig. 5. Fuzzy Set: [5a] Fuzzy Set Properties; [5b] Linguistic Terms of Linguistic Variable

In regards to the FCR's, they are depicted by fuzzy rule-bases. A fuzzy rule establishes that: If the cause concept owns the linguistic term identified in the antecedent; then the effect concept is instantiated with the fuzzy value of the consequent. What is more, the membership degree of the antecedent fuzzy set is attached to the consequent one. A FCR between two concepts is fully stated when: For each linguistic term owned by the antecedent concept there is just one fuzzy rule. Thus, the set of fuzzy rules for the $FCR_{c\text{-}e}$ between two concepts, $c \rightarrow e$, is organized into a fuzzy rule-base ($FRB_{c\text{-}e}$). A FCR does not estimate the real value for the concept as it does a derivate; only it express the qualitative perturbation that supposes will occur. Also, an *accumulative* effect is achieved on a given concept when it is biased simultaneously by several antecedent concepts. So a FCA shifts the fuzzy set of the consequent concept towards the direction of causal bias over its UoD.

On the matter of FIR's, these relationships impose a variation or level value on the state of a consequent concept. A FIR is fully stated by a rule base that defines one fuzzy rule for each linguistic term, or combination of fuzzy values, owned by the antecedent concept(s). According with the linguistic term(s) that hold(s) the antecedent concept(s) in a given time, a specific linguistic value is assigned to the consequent concept. Moreover, an *aggregative* effect is applied on a specific concept when it is simultaneously instantiated with the same linguistic term by several FIR's. This reinforcement event adds the membership degree of the effect concept over the y-axis. Generally, this type of fuzzy inference is brought out by classic methods as Max-Dot and Max-Min.

In relation to the inference mechanism, its mathematical baseline is widely demonstrated by Carvalho (2001). However, in this section the baseline is summarized as follows: The fuzzy causal inference grows from a Causal Output Set (COS) and a Variable Output Set (VOS). Firstly, for each FCR between a couple of concepts, $c \rightarrow e$, is carried out a $COS_{c\text{-}e}$. Therefore, a $COS_{c\text{-}e}$ is outcome from one or a maximum of two fuzzy rules that simultaneously fire. When just one rule fires, the consequent fuzzy set, stated in the rule, becomes the $COS_{c\text{-}e}$ with a membership degree of 1.

But, when two rules fire, the COS_{c-e} is grown from the involved consequent fuzzy values. If both consequents represent the same linguistic term, as in Fig. 6^a, the two consequent fuzzy sets are added into a single COS_{c-e} with a membership degree of 1, as is shown in Fig. 6^b. Otherwise, the consequent fuzzy sets are different, see Fig. 6^c, so they are turned into a result set (U), as in Fig. 6^d, by the Max-Dot method. Next, the result set U_{c-e} is transformed into a COS_{c-e}, just like in Fig. 6^e.

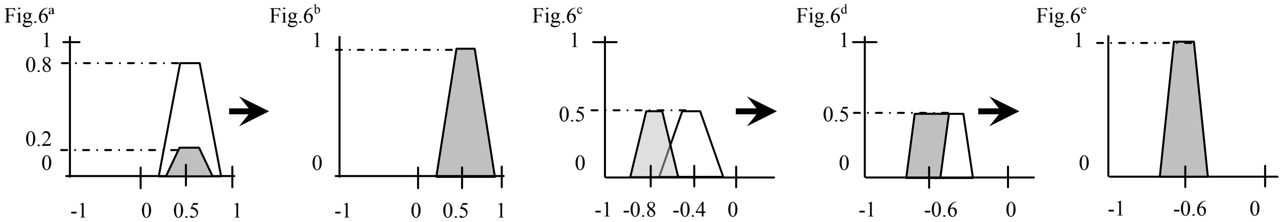


Fig. 6. COS outcome: [6^a] Same fuzzy values; [6^b] New COS; [6^c] Different fuzzy values; [6^d] Result set U; [6^e] New COS

Secondly, a VOS_e is fulfilled based upon the number of FCR's that simultaneously arrive to the consequent concept *e*. Thus, two sceneries are possible: When only just one antecedent concept *c* biases *e*, the COS_{c-e} becomes the VOS_{c-e}. Otherwise, there are several antecedent concepts, c^i: c^1, c^2, .., c^n, that exert *e*. Therefore, the final VOS_e is the result of successive accumulations of the sets $COS_c{}^i{}_{-e}$. This process initializes VOS_e with the first COS_{c1-e}. Next, it takes up a loop to process the remaining $COS_c{}^{2..n}{}_{-e}$. At each step *i*, the $COS_c{}^i{}_{-e}$ is compared with the current VOS_e, as in Fig. 7^a. The fuzzy set with less variation is *shifted* towards the one with the greatest change. Moreover, it is added an extra area for describing the *carrying* of the causal accumulation, this effect is sketched with a dotted line in Fig. 7^b.

As a consequence of the successive accumulation of the sets $COS_c{}^i{}_{-e}$ over the VOS_e, it is necessary to take over the possible saturation of the VOS_e. This issue happens when the support set of the current VOS_e is extended far away of the limit for the x-axis, -1 or 1, as occurs in Fig. 7^c. So it is estimated the length of the overflow in order to cut away the corresponding area. As a result, the exterior declination is aligned with the point that corresponds to the limit. Also, the interior declination is moved towards the limit a proportion equivalent to the surplus length, as is illustrated in Fig. 7^d. Therefore, an area alike the leftover area is taken off in the internal side of the current VOS_e. Finally, the VOS_e brought out the process of the last $COS_c{}^n{}_{-e}$ becomes the fuzzy set assigned as the linguistic term to the consequent concept *e*.

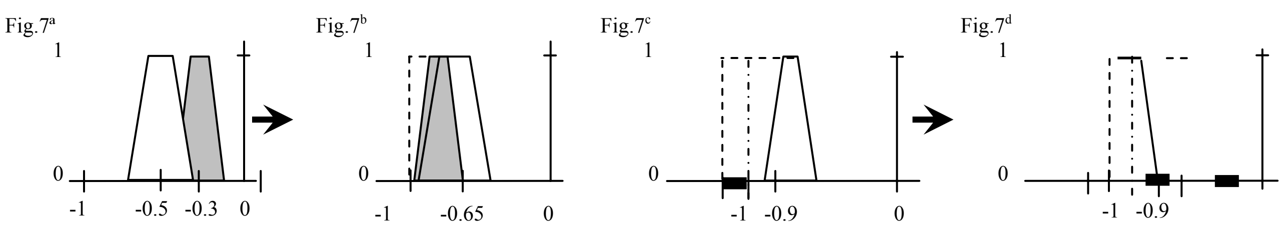


Fig. 7. VOS estimation: [7^a] Comparison COS_{i-e} against $VOS_{(i-1)-e}$; [7^b] Shift the fuzzy set with less variation to achieve the new VOS_{i-e}; [7^c] Identification the exceeding area to the x-axis' limit; [7^d] Elimination of the exceeding area of both sides of the VOS_{i-e};

Basically, a FCA between two fuzzy sets *M* and *N* is stated in equation (7) as *M@N*, where, (M, $N \in F(X)$), and *X* is a discrete interval between [0, 1]. This interval can be extended away the limit 1, whether saturation occurs. Also, sets *M* and *N* depict positive variations VOS's$^+$. But the variation stated by *N* is lesser or equal than the one pointed out by *M*. So prior taking over the saturation event, x_i is a discrete point that acquires values from 0 until the maximum value of the support set of the VOS. What is more, the FCR equation is supported by two functions: Shift and carry. The shift function estimates in (8) the final point in x-axis where *N* will be displaced towards *M*. This value grows from the difference between the minimal value of the core set attached to *M*, and the minimal value of the support set of *N*. In regards with the carry function, its equation is set in (9). The carry function brings out the accumulation effect in each point x_i of UoD's x-axis.

$$\mu_{M@N}(x_i) = \min\{1, \mu_M(x_i) + \mu_N(x_{i-shiftN}) + carry(x_{i-1})\}. \tag{7}$$

$$shift\ \ N = \min(M_i) - \min(Support\ \ N). \tag{8}$$

$$carry(x_i) = \max\{0, \mu_M(x_i) + \mu_N(x_{i-shiftN}) - 1\},\ and\ \ carry(x_{-1}) = 0. \tag{9}$$

5.2 Case of Study

As concerns the Student Model application introduced in section 3.2, we put in to develop the version based upon RB-FCM's. Firstly, in Fig. 8 is sketched the early CM, with the acronym *rb* and a consecutive number as the label for the causal relationships; its meaning corresponds to the rule base number that states the fuzzy rule base corresponding to the FCR. Also, in Table 8 is given away the adjacency matrix with the number of the rule bases in order to make easy the identification of each FCR. What is more, the initial states values attached to the concepts are brought out in Table 9[a], where appear the *variation* value that corresponds to ten linguistic variables. Those initial variation values grown from the causal influences that it is believed that occur when the student takes his/her first lecture for Object Oriented learning. Thus, these kind of suppositions are stated as initial linguistic terms, e.g., it is believed that his/her background in Java decrease much as result of some missconceptions, and his/her aims at encapsulate methods increase quite few. Moreover, an instance of a RB is outlined in Table 9[b]. Such table corresponds to the FCR between the antecedent concept *miss of mathematical reasoning* and the consequent concept *dislike recursive functions*. In the left column are stated the linguistic terms attached to the antecedent concept, whilst in the right column are brought out the fuzzy values for the consequent concept.

Table 8. Fuzzy adjacency matrix for the RB-FCM in Fig. 8

A	a	b	c	d	e	f	g	h	i	j
a		1	2	3	4					
b	5			6		7		8	9	
c					10					
d			11							
e							12			
f							13			
g			14	15	16	17		18	19	20
h									21	
i								22		23
j							24			

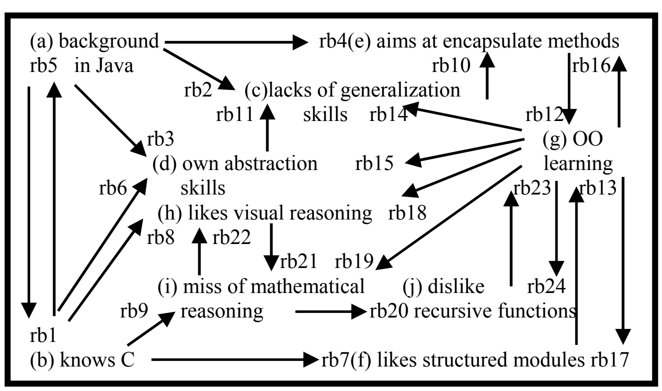


Fig. 8. RB-FCM with a holistic view from Student's and Coach's agents

Table 9. Definition of the RB-FCM: [9[a]] Vector of initial states; [9[b]] Rule base for the FCR20

9[a]	Concept	Initial Linguistic Term
a	background in Java	decrease much
b	knows C	increase medium
c	lacks of generalization	increase quite few
d	own abstraction skills	decrease few
e	aims at encapsulate methods	increase quite few
f	likes structured modules	increase few
g	OO learning	increase quite few
h	likes visual reasoning	increase much
i	miss of mathematical reasoning	decrease quite few
j	dislike recursive functions	increase medium

9[b]	miss of mathematical reasoning	dislike recursive functions
1	If increase too much	Then increase much
2	If increase much	Then increase medium
3	If increase medium	Then increase few
4	If increase few	Then increase quite few
5	If increase quite few	Then maintains
6	If maintains	Then maintains
7	If decrease quite few	Then decrease few
8	If decrease few	Then decrease medium
9	If decrease medium	Then decrease much
10	If decrease much	Then decrease too much
11	If decrease too much	Then decrease too much

Based on a plenty of equations like the (7) to (9), the RB-FCM engine fulfills the causal behavior. Along the simulation fuzzy-causal inferences are achieved. Prior to start the process, the concepts' state values are initialized according with the linguistic terms stated in Table 9[a]. Next, it takes up successive increments of discrete time *t* are done. At each step, the causal effects that simultaneously exert the concepts are accumulated according to the causal paths sketched in Fig. 8.

Thus, new concepts' state values are computed over the UoD of the linguistic variables through the execution of the FCA's. As a result, a kind of histogram, like the one pictured in Fig. 9, is accomplished along the cycles. The graph shows the evolution of the concepts' state values during several iterations. In the y-axis appears the normalization values for the concepts in the range [-1, 1]; whilst, in the x-axis are given away the number of the cycles. Furthermore, the state value owned for a given concept in an instant *t* is sketched by a dot.

The image for the whole RB-FCM in any time *t* is grown from the appreciation of the values that all the concepts own in *t*. The behavior of any concept is stemmed after compare its values along several cycles. During a while, it is evidenced the succession of increments–decrements of the concept's state values, which represent the causal behavior. So it is possible to appreciate several behavior patterns characterized by ups and downs, as the case of the concept *d*, own abstraction skills.

Thus, it is possible to appreciate several behavior patterns characterized by ups and downs, as the case of the concept *d*, own abstraction skills. What is more, some concepts, as *background in Java and learning OO*, bring out ascendant inertias. Although, in this particular case there are not concepts that hold their initial stated because all of them receive at least one causal influence.

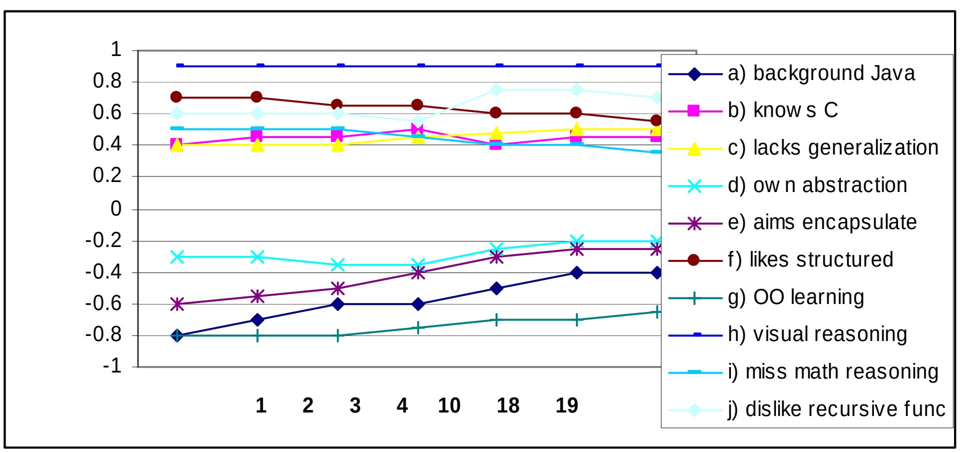


Fig. 9. Concepts' State Histogram. The graphic points out the evolution of the concept's states

The process breaks down when the fuzzy values meet a fixed value. Thus, in Fig. 10 appears the final value reached for the ten concepts that compound the CM. Wherefore, it is revealed that the background in Java is less deficient than its initial value. Due to, it starts with decrease much, and it ends with a normalized value of -0.4, that corresponds to decrease few.

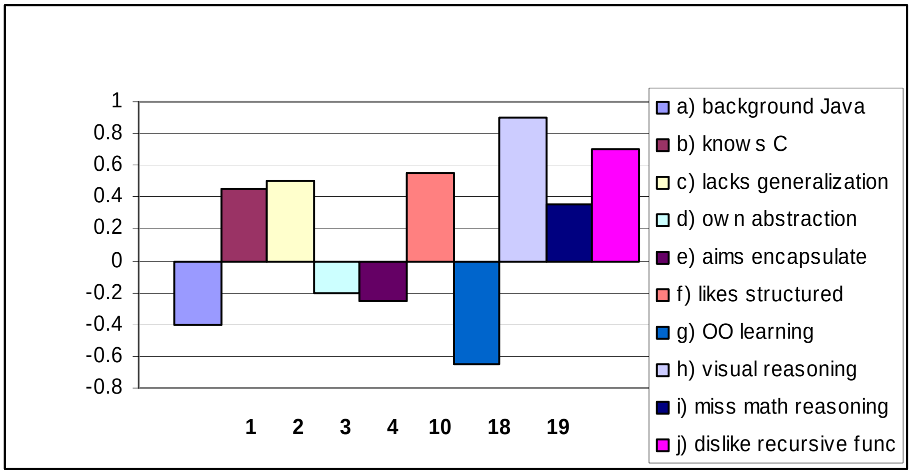


Fig. 10. Stability. It is the region where a fixed value for all the concepts is found

The behavior of the concepts, and the whole RB-FCM, are given away from two analyses: 1) Transformation upon the concepts' state values during the iterations. 2) The final state reached by the concepts' states values. As a result of this kind of interpretation, the prediction about the causal behavior and outcome are done. Wherefore, based on this CM the Student Model is able to anticipate the casual effects that a given teaching-learning experience yields on the student.

6 Discussion and Further Work

The CM's stated in this paper are a paradigm for giving away the causal phenomenon. They are acknowledged as *cognitive*, due to bring out a way of thought of the individuals. CM's are a tool devoted to externalize how a person understands a given issue. They make easy the analysis of a subject domain in order to explain earlier results since the cause-effect view.

Moreover, CM's support the process for making decisions due to they stem causal predictions about the behavior and outcomes of the problem to solve. Wherefore, a CM is a graphical representation regarding to the mental model that an individual owns about a real matter.

CM's are an alternative approach for modeling qualitative systems dynamics as the one proposed by Forrester (1968). Because of they sketch workflows and feedbacks between the entities of a system through cause-effect relationships. Also, CM's achieve qualitative processing instead of using quantitative methods that some times result complex and inaccurate.

In regards to the qualitative version of CM's, this model focuses on the analysis of the causal relationships exclusively. In this approach, the state of the concepts is not specified exactly, nor the changes that occur on them. The key point is to estimate the type of influence that bias on a given concept at iteration t. Wherefore, the simulation exclusively remains on the adjacency matrix manipulation. Thus, the interpretation of the causal behavior of the CM is grown from the values stored in the different versions of A^t. Nevertheless, qualitative CM's lack of precise meaning nor a sound treatment of causal relations. In addition, the formal model owns some pitfalls in the multiplication operation for the causal value a. Due to the laws of multiplication are inconsistent for cases as: $a * \oplus$, *a* * Θ, and *a* * *?*. So alternative paradigms, as the relational model proposed by Chaib (2002), are worthy to be considered.

On the matter of fuzzy CM's, they are the most spread version of CM's. Whereby, their applications are heterogeneous as the radiation therapy systems, and the study of political-economical issues in Cyprus. The main contribution of the fuzzy CM's is the attempt to introduce the fuzzy logic to the CM's arena. But, this intention became the major pitfall for this kind of CM's. Because of the former linguistic representation of the concept's states and relation values is lost as a result of the "fuzzification" process. Wherein, the whole mathematical simulation is exclusively brought out numerically. During, the simulation the fuzzy engine never applies a defuzzification process nor it does a classic fuzzy inference. The early goal to deal with linguistic terms stemmed from the natural language is not accomplished. Furthermore, other drawback of the fuzzy CM's is the lack of a method for estimate stability conditions that could happen during the simulation process.

Regarding RB-FCM's their virtue is to provide a meeting point where fuzzy logic and CM's collaborate to model causal phenomena. This version retakes the former ideal of the fuzzy CM's in order to manipulate linguistic terms with a sound baseline. RB-FCM's are a paradigm for dealing with qualitative knowledge that allows to state concepts and relations through natural language terms. Also, they offer a dual engine for achieving fuzzy inferences according with the kind of relationships, causal or influence. However, the cognitive mapping is more complex than the other versions. Moreover, the inference produces overhead. Furthermore, the approach does not mix opposite effects, negative with positive, at the generation of the final VOS_e.

As a workline to carry on with the research of the CM's, we propose: The study of the natural methods that explain how the individual chooses the concepts and the causal relations involved in a CM. Also, the analysis of the intuitive procedures that reveal how is accomplished the causal reasoning in the mind. Moreover, it is required to go forward methods that elicit individual's causal beliefs, algorithms that outcome automatically CM's, and guidelines to set criteria and procedures to validate the predictions given by the CM's. Finally, it is desirable some enhancements for the CM's, as: A sound theory that is well founded, specialized tools that make easy the development of applications, mechanism for gathering evidences that support the predictions and a holistic model to take advantage of the strengths of the versions.

Particularly, as further work to be done is considered: The organization of an ontology about the teaching-learning experiences, the development of a predictive Student Model based upon Cognitive Maps, the experimental use of the Cognitive Maps into the Web-based Education Systems, and the recopilation of empirical data to be studied by analysis of variance and covariance upon independent, dependent and nuisance variables [Chin, 2001].

Besides the early stated philosophical principle, the causal phenomenon owns some grounds given away from physical and spiritual sources, as the cause-effect Newton's law, and the Biblical verse that claims: Be not deceived, God can not be not mocked; for whatsoever a man soweth, that he also reap [Galatians, 6.7].

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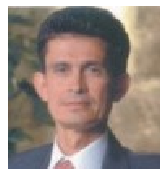

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