A Divide-and-Conquer Approach to Commercial Territory Design

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Abstract. A new heuristic procedure for a commercial territory design problem is introduced in this work. The proposed procedure is based on the divide-and-conquer paradigm and basically consists of a successive dichotomy process on a given large instance of the problem. During this process, a series of integer quadratic subproblems is solved. The obtained computational results have shown that the proposed heuristic is an attractive technique for obtaining locally optimal solutions for large instances which are intractable by using exact optimization methods.

Keywords. Territory design, heuristic optimization, integer quadratic programming, divide-and-conquer approach.

Procedimiento divide y vencerás para el diseño de territorios comerciales

Resumen. En este trabajo se presenta un procedimiento heurístico para el diseño de territorios comerciales. El procedimiento propuesto, basado en el paradigma dividir-y-vencer, consiste básicamente en un proceso de dicotomías sucesivas a partir de una instancia dada. Durante este proceso se resuelven una serie de subproblemas de programación cuadrática entera. Los resultados computacionales muestran que la heurística propuesta es una técnica de solución atractiva que permite la obtención de soluciones óptimas locales para instancias grandes del problema, las cuales resultan intratables al intentar resolverlas a través de métodos exactos.

Palabras clave. Diseño territorial, optimización heurística, programación cuadrática entera, procedimiento divide y vencerás.

1 Introduction

The problem addressed in this work is motivated by a real-world application from a beverage distribution firm in the city of Monterrey, Mexico. The problem consists of finding a partition of the entire set of city blocks (basic units, BUs) into p territories, such that a measure of territory compactness is maximized. Additionally, it is required to find territories that are connected and balanced (similar in size) with respect to the number of customers and the product demand. A territory is connected if the set of BUs belonging to it induces a connected subgraph.

This problem can be found in the distribution firm before the routing plan takes place. Having shorter routes in product distribution is a direct consequence of having compact territories in the design stage. In addition, it is well established by the firm that compact territories reduce the number of unsatisfied customers caused by different deals offered to their customers.

The first related work which appeared in the literature is the one done by Ríos-Mercado and Fernández [21]. In this work, a reactive GRASP procedure is developed in order to minimize a dispersion measure (based on the *p*-center problem objective) subject to multiple balancing constraints (number of customers, product demand, and workload). Caballero-Hernández *et al*. [6] studied a related model by considering BU joint-assignment constraints. They developed a

GRASP including a pre-processing phase that first satisfies the joint-assignment constraints and then a construction phase based on a territory merging mechanism with relatively good results.

Salazar-Aguilar *et al*. [23] present an exact optimization framework for solving small- to medium-size instances of the problem. This method is successfully applied to both *p*-median and *p*-center objective models. In addition, the authors propose new integer quadratic programming models which allow to efficiently solve large instances by commercial MINLP solvers such as DICOPT and AlphaECP. Their reported results motivated the solution procedure suggested in this work.

In this paper, we propose a divide-and-conquer heuristic aiming at solving large instances of the commercial territory design problem based on the *p*-median objective for measuring dispersion. This work can be seen as an extension of the work by Salazar-Aguilar *et al*. [23] focusing on exact methods for small- and medium-size instances of the problem.

In particular, our proposed heuristic follows a successive dichotomy idea, where at each iteration a given subproblem is partitioned into two smaller subproblems by solving an associated territory design problem with two territories. When a given subproblem is small enough, it is solved exactly by means of an integer quadratic programming model.

The proposed procedure (IQPHTDP) was evaluated over a set of randomly generated instances based on real-world data. The results revealed that IQPHTDP is a very attractive technique that allows obtaining good quality solutions for large instances in reasonable time.

The remainder of this paper is organized as follows. Section 2 is devoted to the description of the problem. Section 3 highlights relevant works in the territory design/districting literature. The proposed procedure is described in Section 4. Our computational results are presented in Section 5, followed by conclusions in Section 6.

2 Problem Statement

Territory design or districting consists of dividing a set of basic units (typically city blocks, zip-codes or individual customers) into subsets or groups according to specific planning criteria. These groups are known as territories or districts. Diverse applications from different areas require the creation of territories, for instance, school districts, political districting, and sales territory design (see Kalcsics *et al*. [15]). There are a few works related to the commercial territory design problem. The first work related to this problem was introduced by Ríos-Mercado and Fernández [22]. Different versions of this problem have been studied by Caballero-Hernández *et al*. [6] and Salazar-Aguilar *et al*. [23]. Specifically, the problem is formulated as follows. A firm wants to partition the basic units (blocks) of the city into a specific number of disjoint territories that are suitable according to their logistic, marketing and planning requirements. The company wishes to create a specific number of territories (p) that are balanced with respect to each of the two attributes, namely, the number of customers and the product demand. Additionally, each territory needs to be connected, so that the basic units (BUs) in the same territory can reach each other without leaving the territory. Territory compactness is required to guarantee that customers within a territory are relatively close to each other. The problem is modeled by a graph $G=(V,E)$, where V is the set of nodes (city blocks) and E is the set of edges that represents adjacency between blocks. That is, a block or BU j is associated with a node, and an edge connecting nodes i and j exists if BUs i and j are located in adjacent blocks. Multiple attributes such as geographical coordinates (c_x,c_y), the number of customers and the product demand are associated to each node $j \in V$. It is required that each node is assigned to only one territory (exclusive assignment). In particular, the firm seeks a perfect balance among territories; it means that each territory must have about the same number of customers and product demand associated. Let $A=\{1,2\}$ be the set of node activities, where 1 refers to the number of customers and 2 refers to the product demand. We define the size of territory V with respect to activity a as $w^a(V_k)=\sum_{i\in V_k}(w_i^a)$, where $a\in A$ and w_i^a is the value associated to activity a in the node $i\in V_k$. Hence, the target value is given by

$\mu^a = \sum_{i \in V} \frac{w_i^a}{p}$. Another important constraint is connectivity, i.e., for each pair of nodes *i*, *j* belonging to the same territory, there must exist a path between them such that it is totally contained in the territory. In addition, in each territory the BUs must be relatively close to each other (compactness).

Depending on how the dispersion is measured, different models can be obtained. In this work we consider a dispersion measure based on the *p*-median problem. A full description of this model can be found in Salazar-Aguilar *et al.* [23]. For completeness, here we include the combinatorial formulation of the MPTDP model studied in this work. Let Π be the set of all possible *p*-partitions of *V*. For a particular territory, $B_k, c(k)$ is a territory center and d_{ij} is the Euclidian distance between nodes *i* and *j*; $i, j \in B_k$. A territory center is computed as

$$c(k) = arg \min_{i \in B_k} \sum_{j \in B_k} d_{ij}.$$

$$\text{(MPTDP)} \quad \min_{B \in \Pi} \quad f(B) = \sum_{k=1,\dots,p} \sum_{i \in B_k} d_{ic(k)} \quad (1)$$

subject to:

$$w^a(B_k) \in [(1-\tau^a)\mu^a, (1+\tau^a)\mu^a], \quad a \in A; k = 1,\dots,p \quad (2)$$

$$G = (B_k, E(B_k)) \text{ is connected}, \quad k = 1,\dots, p \quad (3)$$

In this model, the objective is to find a *p*-partition of *V* such that the dispersion (1) on each territory B_k is minimized. Constraints (2) establish that the territory size (the number of customers and the product demand) should be within the range allowed by the tolerance parameter τ. In addition, each territory should induce a connected subgraph (3). It was shown by Salazar-Aguilar *et al*. [24] that MDTDP is NP-hard. Furthermore, as shown in another paper of Salazar-Aguilar *et al*. [23], there are two mathematical programming models for this problem. In our solution procedure, we make use of the quadratic integer programming (IQP) model introduced in Salazar-Aguilar *et al*. [23], since it was shown to allow an optimal solution of instances with up to 400-500 BUs. When using the linear model, the size of the instances that could be optimally solved is within the range of 250 BUs.

3 Related Work

Districting problems are similar to clustering problems in the sense that both seek to find suitable partitions of the problem; however, there are fundamental differences that make clustering methods not applicable to districting problems. For instance, the presence of balancing and connectivity constraints makes districting problems unique in this regard. For an extensive survey on clustering methods, the reader is referred to the work of Xu and Wunsch [27]. There is also commercial software available such as TerrAlign (http://www.terralign.com) and AlignStar (http://www.alignstar.com/); however, this software is limited to handling sales force deployment in territory design with different objective and planning requirement measures, and therefore cannot be used in our particular districting application.

Table 1 contains a summary of the most important work on territory design that has been developed in diverse fields such as political districting, sales districting, and public services. This table lists the main features included in these applications. The planning criteria (third column), i.e., balancing, connectivity, and a fixed number of territories, are abbreviated as 'B', 'C', and 'F', respectively. For the works in which the number of territories is not fixed, the capital letter 'F' is replaced by 'V', and '-' appears in the cases where connectivity is not a constraint. In the fourth column, 'Single(∑)' means that two or more criteria were placed together in a weighted sum objective function.

This survey reveals that there are only a few works addressing the commercial territory design problem. Furthermore, among those works, the only one studying the *p*-median based dispersion measures focuses on exact methods for small-

Table 1. Summary of territory design applications

Author	Application	Criteria	Objective	Solution Technique
Hess and Weaver (1965)	Political	B,C,F	Single	Location-allocation
Garfinkel and Nemhauser (1970)	Political	B,C,F	Single	Exact procedure
Hess and Samuels (1971)	Sales	B,-,F	Single	Location-allocation
Bertolazzi et al. (1977)	Services	B,-,F	Single	Exact procedure
Marlin (1981)	Services	B,-,F	Single	Location-allocation
Pezzella et al. (1981)	Services	B,C,F	Single	Location-allocation
Fleischman and Paraschis (1988)	Sales	B,-,F	Single	Location-allocation
Hojati (1996)	Political	B,C,F	Single	Location-allocation
Mehrotra (1998)	Political	B,C,V	Single	Heuristic based on Branch and Price
Drexl and Haase (1999)	Sales	B,C,V	Single	Heuristic
Guo et al. (2000)	Political	B,C,F	Bi-objective	MOZART
Muyldermans et al. (2002)	Services	B,C,F	Single(∑)	Heuristic of two phases
Blais et al. (2003)	Services	B,C,F	Single(∑)	Tabu search
Bozkaya et al. (2003)	Political	B,C,F	Single(∑)	Tabu search and adaptive memory
Ricca and Simeone (2004)	Political	B,C,F	Single(∑)	Old bachelor acceptacnce
Bong and Wang (2004)	Political	B,C,F	Three-objective	Tabu search and scatter search
Baçao et al. (2005)	Political	B,C,F	Single	Genetic algorithms
Chou et al. (2007)	Political	B,C,F	Single(∑)	Simulated annealing and genetic algorithms
Tavares and Figueira (2007)	Services	B,C,F	Bi-objective	Evolutionary algorithm with local search
Caballero-Hernández et al. (2007)	Commercial	B,C,F	Single	GRASP
Segura-Ramiro et al. (2007)	Commercial	B,C,F	Single	Location-allocation
Ricca and Simeone (2008)	Political	B,C,F	Single(∑)	Descent, tabu search old bachelor acceptance, and simulated annealing
Ríos-Mercado and Fernández (2009)	Commercial	B,C,F	Single	Reactive GRASP
Salazar-Aguilar et al. (2011)	Commercial	B,C,F	Single	Exact procedure

and medium-size instances. Therefore, the contribution of our work is to present a heuristic for solving large instances of the commercial TDP with p-median based objective function.

4 Proposed Divide-and-Conquer Procedure

The main idea behind the proposed procedure is to decompose the problem (or subproblem) into two smaller subproblems by solving a TDP model with $p = 2$ super-territories. This stems from the fact that solving a TDP with $p = 2$ is considerably easier to solve than solving a TDP with a large value of p. When building this subproblem with $p = 2$, special attention must be paid to the way the tolerance parameter for the balancing constraints is chosen.

Recall that a feasible design is one which presents imbalances within τ^a percent from the target value μ^a. If this value were to be used in the subproblems, the error would accumulate yielding infeasible designs. This motivates the introduction of a control parameter ρ whose main role is to adjust the tolerance level in the subproblems aiming at yielding feasible designs as output. This parameter is typically fine-tuned empirically. This 2-partition procedure is iteratively performed to create subproblems of smaller size with respect to the number of BUs. When this number of BUs for a given subproblem is smaller than a user-specified threshold *maxN*, the subproblem is no longer 2-partitioned, but solved optimally with an appropriate value of ρ. As stated before, a reasonable value for *maxN* is 300.

Algorithm 1 shows the proposed solution procedure in pseudocode. The algorithm takes as input a problem instance *I*. Note that when solving a subproblem by means of *SOLVE*(V_c,p_c), a p_c-*partition* $S_c = (S_1, \dots, S_p)$ is sought and the balancing constraints are adjusted as follows:

Algorithm 1. IQPHTDP(*I*,*maxN*, ρ)

Require:
 $I = I(V,p)$:= Instance of TDP, where V is the set of BUs and p is the number of territories
 $maxN$:= Threshold on BUs for solving the subproblems
 ρ := Control parameter for adjusting the range of the balance constraints
Ensure: $S = (V_1, \dots, V_p)$:= A p-partition of V
 $S \leftarrow \emptyset$
 $I_0(V,p) \leftarrow I(V,p)$ {Original instance}
 $L \leftarrow \{I_0\}$ {Subproblem list}
 while ($L \neq \emptyset$) **do**
  $I_c(V_c,p_c) \leftarrow POP(L)$ {Remove instance from L}
  if ($|V_c| \leq maxN$) **then** {Solve the subproblem}
   $S_c = (S_1, \dots, S_{p_c}) \leftarrow SOLVE(V_c, p_c)$
   $S \leftarrow S \cup S_c$ {Add partition to solution set}
  else {Partition the subproblem into 2 subproblems}
   $S_c = (S_1, S_2) \leftarrow SOLVE(V_c, 2)$
   $p_1 \leftarrow \lceil \frac{p_c}{2} \rceil$
   $p_2 \leftarrow p_c - p_1$
   $L \leftarrow L \cup \{I(S_1,p_1), I(S_2,p_2)\}$ {Add the two new subproblems to L}
  end if
 end while
 return $S = (V_1, \dots, V_p)$

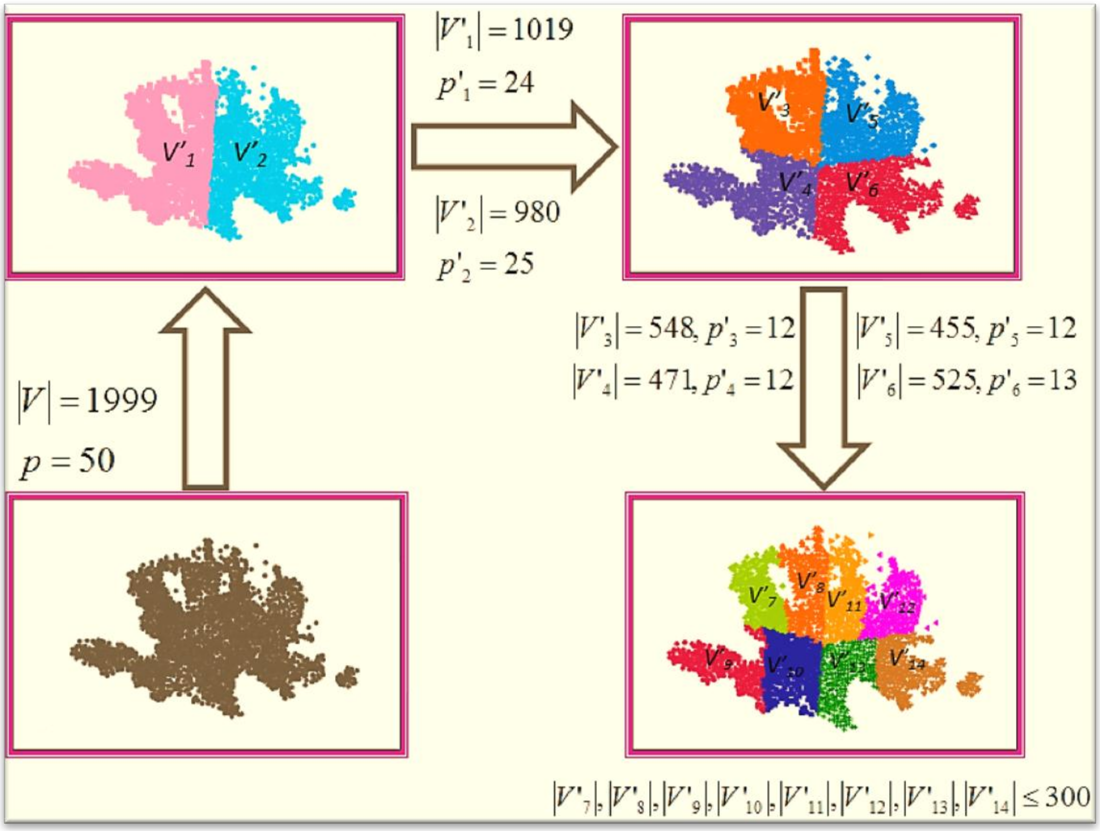

Fig. 1. Successive dichotomy process for solving instance

$(1-\rho\tau^a)\mu_c^{(a)} \leq \sum_{j\in S_k} w_j^a \leq (1+\rho\tau^a)\mu_c^{(a)}$, where the target $\mu_c^{(a)}$ is computed as $\mu_c^{(a)} = \frac{1}{p_c}\sum_{i\in V_c} w_i^a$. The control parameter ρ should be fine-tuned Typical values are within the [0.1, 0.5] range. It helps to keep balanced partitions as much as possible and this is required because if the initial dichotomy produces a 2-partition with high relative deviation with respect to the average (target value), in the following dichotomy this value carries an aggregated effect that may render some unbalanced territories at the end.

Computational complexity. The number of subproblems solved by IQPHTDP for an instance of size (n,p) is bounded by $O(\frac{2^{\alpha+1}-1}{\alpha-1})$, where $\alpha = \log_2\lceil n/maxN\rceil$. Now, each subproblem requires solving an IQP which is basically an enumerative procedure such as branch and bound that has an exponential worst-case theoretical bound. However in practice, relatively large instances can be handled. For instance, consider an instance of size (2000, 40), then IQPHTDP would solve 1 subproblem of size (2000, 2), two subproblems of size (1000, 2), four subproblems of size (500, 2) and eight subproblems of size (250, 5), that is 15 subproblems. Each one takes from 1 minute up to 30 minutes, and the subproblems with $p > 2$ are most time consuming. We should point out that attempting to solve directly an instance of size (2000, 40) by IQP is useless.

An Illustrative Example. Suppose that IQPHTDP is used for solving an instance *I* with (n,p)=(1999,50) and input parameters *maxN*=300 and $\rho = 0.8$. Fig. 1 shows the dichotomy process. Note that in the first dichotomy each partition V'_1

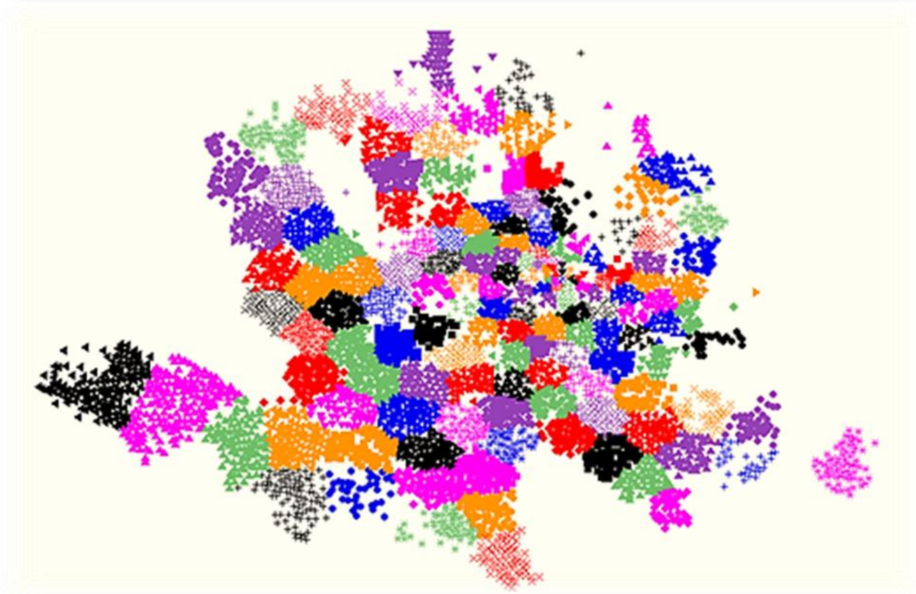

Fig. 2. Final solution for instance *I* (using IQPHTDP)

and V_2' contains half the total number of required territories (thus 25 out of 50), and the number of BUs on each of them is greater than *maxN*, thus another dichotomy is needed. Partitions V_1' and V_2' are used to generate two subproblems of TDP ($[G_1' = (V_1', E(V_1'))] \subset G$ and $[G_2' = (V_2', E(V_2'))] \subset G$, respectively) which are solved using $p = 2$. In Fig. 1, (V_3', V_4') corresponds to the 2-partition of V_1', and (V_5', V_6') is a 2-partition of V_2'. Partitions V_3', V_4', V_5' and V_6' contain more BUs than allowed by *maxN*, so the dichotomy process is applied on each of them until the last obtained partitions $V_l': l = 7, \ldots, 14$ contain less BUs than the limit value (given by *maxN*). The latter are solved using the number of territories contained in each partition. For instance, the subproblem given by V_7' is solved for $p_7' = 6$, and the subproblem given by V_8' is solved for $p_8' = 6$. The upper and lower balancing requirements are taken from the original instance *I*. Note that the balancing requirements for dichotomy are computed using the control parameter ρ and the number of territories contained on each sub-instance (see Algorithm 1). The final solution for the instance *I* is computed by putting together all partitions obtained by solving the small subproblems (in the example, the small subproblems are those generated by $V_l': l = 7, \ldots, 14$. Fig. 2 shows the final solution obtained for the instance *I* by applying IQPHTDP. Note that some small subproblems may be infeasible with respect to the balancing constraints, so the solution for the original instance will be infeasible.

5 Experimental Work

The procedure was coded in C++ and compiled with the Sun C++ compiler workshop 8.0 under the Solaris 9 operating system, and run on a SunFire V440. Each integer quadratic subproblem is solved by calling GAMS/DICOPT MINLP solver. The data sets were taken from the library developed by Ríos-Mercado and Fernández (2009). These data sets contain randomly generated instances based on real world data provided by the firm. The number of customers and product demand are generated from distributions based on historical data. The experimental work was carried out over two instance sets $(n, p) \in \{(1000{,}50), (2000{,}50)\}$ with

Table 2. Best dispersion values (p-median) for instances from set (2000, 50)

Instance	$\rho = 1.0$	$\rho = 0.1$	$\rho = 0.2$
DU2k-1	Infeas	Infeas	54423.02
DU2k-2	Infeas	54337.56	54487.95
DU2k-3	Infeas	Infeas	55111.29
DU2k-4	Infeas	55642.04	54963.38
DU2k-5	Infeas	54616.84	55122.05
DU2k-6	Infeas	54145.92	55070.89
DU2k-7	Infeas	54813.34	Infeas
DU2k-8	Infeas	53048.47	54722.55
DU2k-9	Infeas	54968.87	55402.97
DU2k-10	Infeas	Infeas	55085.06

Table 3. Best dispersion values for instances from set (1000, 50)

Instance	$\rho = 1.0$	$\rho = 0.1$
DU1k-1	Infeas	25679.38
DU1k-2	Infeas	26455.53
DU1k-3	Infeas	25965.95
DU1k-4	Infeas	26286.99
DU1k-5	Infeas	26522.25
DU1k-6	Infeas	26180.19
DU1k-7	Infeas	26325.41
DU1k-8	Infeas	27022.62
DU1k-9	Infeas	26347.22
DU1k-10	Infeas	26896.69

$\tau^a = 0.05$. For each of them, 10 instances were generated. Different values of ρ were used in order to determine the effect produced by this parameter in the final solution reported by the IQPHTDP procedure.

In Table 2, the first column contains the instance name; each of the following columns show the objective value reported by the IQPHTDP for $\rho \in \{1.0, 0.1, 0.2\}$. An appropriate selection of parameter ρ is very important for the success of the proposed procedure. If $\rho = 1.0$, it means that the balancing deviation in all IQP subproblems is given by τ^a. This implies that, when the size of a partition is really close to the balancing bounds, subsequent partitions created from this partition may be very unbalanced with respect to the target value in the original instance. Hence, the final solution reported by IQPHTDP is

Table 4. Comparison between IQPHTDP and GRASP-RF, instances from set (1000, 50)

Instance	***p*-median**		***p*-center**	
	IQPHTDP	**GRASP-RF**	**IQPHTDP**	**GRASP-RF**
DU2K-01	25679.38	31541.49	71.89	74.68
DU2K-02	26455.53	30289.81	82.13	69.38
DU2K-03	25965.95	30350.12	73.56	72.77
DU2K-04	26286.99	31084.62	68.1	69.87
DU2K-05	26522.25	30154.66	72.79	67.54
DU2K-06	26180.19	Infeas	68.47	Infeas
DU2K-07	26325.41	29173.25	64.28	71.04
DU2K-08	27022.61	Infeas	69.78	Infeas
DU2K-09	26347.22	30048.23	70.09	67.07
DU2K-10	26896.69	29369.11	77.31	62.17

Table 5. Comparison between IQPHTDP and GRASP-RF, instances from set (2000, 50)

Instance	***p*-median**		***p*-center**	
	IQPHTDP	**GRASP-RF**	**IQPHTDP**	**GRASP-RF**
DU2K-01	54423.02	58909.07	76.69	66.07
DU2K-02	54487.96	61133.65	85.41	63.39
DU2K-03	55111.29	58654.13	75	63.85
DU2K-04	54963.32	58916.57	67.73	62.3
DU2K-05	55122.05	58676.64	67.71	61.15
DU2K-06	55070.89	59558.59	81.36	65.72
DU2K-07	Infeas	62371.46	Infeas	68.38
DU2K-08	54722.55	59908.42	80.83	67.55
DU2K-09	55402.97	58590.57	74.74	66.58
DU2K-10	55085.06	58560.1	77.37	60.55

infeasible with respect to the balance constraints in the original problem. In contrast, if the ρ value is very restrictive, some subproblems cannot be solved with feasibility (see $\rho = 0.1$), then an infeasible solution to the original instance is obtained.

When ρ=0.2 was set, it allowed to solve more instances than $\rho = 0.1$. A similar behavior was observed for those instances with (1000, 50). However for these instances, $\rho = 0.1$ was a good choice for getting feasible solutions, see Table 3.

To the best of our knowledge, there is no heuristic procedure that allows obtaining solutions for the problem addressed in this work. In Ríos-Mercado and Fernández [22], the authors develop a reactive GRASP for the TDP under a *p*-center based objective function.

Even though that heuristic was developed for a different problem (with a different objective function and three balancing constraints rather than two), we have adapted such procedure for using two balancing constraints and measured the quality of the design obtained in terms of the intended TDP with *p*-median objective. We called this modified procedure GRASP-RF. We solved the two instance sets using both IQPHTDP and GRASP-RF. We compared the quality of the designs obtained by each method under the TDP with *p*-median objective. In addition, we also assessed the quality of the solutions found by our method when aimed at solving the other problem, that is, the TDP under *p*-center objective, and compared them with the solutions obtained by GRASP-RF. Tables 4 and 5 show a summary of this test for the two different data sets. In these tables, Column 1 shows the instance name, Columns 2 and 3 show the comparison of the heuristics for the TDP under the *p*-median objective function, which is the problem addressed in this work. As you can observe, the solutions obtained by IQPHTDP are best in 19 out of 20 instances. The only instance where IQPHTDP failed was DU2K-07.

Now, Columns 4 and 5 in Tables 4 and 5 show the comparison between heuristics for the TDP under the *p*-center objective (TDPC). Note that even though GRASP-RF was specifically designed for addressing the TDPC, and therefore in general obtained better solutions for this problem than the ones found by IQPHTDP, our method is still very competitive, helping to find some better solutions in some cases. For example, we observed that for instances from (1000, 50) the GRASP-RF did not report feasible solutions for 2 out of 10 whereas our method did find feasible solutions in all cases. Furthermore, there were 5 out of 10 instances where the solution reported by IQPHTDP was better than the solution obtained by GRASP-RF.

6 Conclusions

In this paper, the commercial districting problem under a *p*-median objective for minimizing territory dispersion was addressed. A novel heuristic procedure based on the divide-and-conquer paradigm called IQPHTDP has been proposed. This procedure allows obtaining locally optimal solutions for large instances (1000 and 2000 BUs) in short time. These instances were intractable by using existing exact methods. However, the performance of this procedure depends on the choice of the control parameter ρ. As we showed in the experimental work, the best ρ value was 0.2 for the instances with 2000 BUs and 0.1 for the instances with 1000 BUs. Bad values of ρ may yield highly infeasible solutions with respect to the balancing requirements. Therefore, when the final solution is infeasible, the IQPHTDP procedure can be applied by using another ρ value; however, this change does not guarantee that the new solution will be feasible, besides, the time increases for each trial-and-error attempt of the ρ value. In addition, the empirical evidence showed that the proposed method consistently outperformed the only existing method available in literature, to the best of our knowledge.

A natural extension of this work could be the derivation of a local search procedure to reach feasibility in those cases where IQPHTDP is not able to find feasible solutions.

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